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K-Regular Domination In Hesitancy Fuzzy Graph

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Abstract

In this paper we define regular domination set and regular dominating number in hesitancy fuzzy graph and investigate some properties and bounds of regular domination number in varioushesitancy fuzzy graphs.

Keywords:Hesitancy Fuzzy graph, regular dominating set and regular dominating number.

INTRODUCTION:

Fuzzy set theory was introduced by Zadeh LA. Most of the real world problems are enormously complex and contain vague data. In order to measure the lack of certainty, extradevelopment to Fuzzy sets was introduced by Torra V and he named it as Hesitant Fuzzy Sets(HFSs). HFSs are encouraged to handle the common trouble that appears in fixing the membership degree of an element from some potential values. This circumstances is rather common in decision making problems too while an professional is asked to assign different degrees of membership to a set of elements {x, y, z,…} in a set A. Frequently problems arise due to uncertain issues and situations hence one is faced with hesitant moments. The investigator had to find ways and means to take the problems and arrive at a solution. Therefore investigators have taken up the learning and application of HFS. HFSs have been extended Xu Z. and Zhu B, from different perspectives such as, both quantitative and qualitative.Hesitant fuzzy sets introduced by Torra in 2010. Pathinathan et.al. Introduced Hesitancy fuzzy graph in 2015 and discussed various properties . Hesitancy Fuzzy Graphs (HFGs) has been applied to capture the common A.Prasanna, M.A.Rifayathali and S.Ismail Mohideen intricacy that occur during a selection of membership degree of an element from some possible values that make one to hesitate.

In this paper we define regular domination set and regular dominating number in hesitancy fuzzy graph and investigate some properties and bounds of regular domination number in varioushesitancy fuzzy graphs.

1. PRELIMINARIES

This section deals the some basic definitions of fuzzy graphs. It is useful to construct the next section.
A Hesitancy fuzzy graph $G(\mathrm{V},\mathrm{E})$, where the vertex set V is a triplet fuzzy functions it is defined by $\mu_1 : V \to [0,1], \nu_1 : V \to [0,1]$ and $\beta_1 : V \to [0,1]$,these functions are called as membership, non-membership and hesitancy of the vertex $v_i \in V$ respectively and $\mu_1(v_i)+\nu_1(v_i)+\beta_1(v_i)=1$, $\beta_1(v_i)=1-[\mu_1(v_i)+\nu_1(v_i)]$. The edge set of $G(\mathrm{V},\mathrm{E})$ is a triplet fuzzy functions it is defined by $\mu_2 : V\times V \to [0,1], \nu_2 : V\times V \to [0,1]$ and $\beta_2 : V\times V \to [0,1]$, such that $\mu_2(\mathrm{uv}) \le \mu_1(\mathrm{u}) \wedge \mu_1(\mathrm{v})$, $\nu_2(\mathrm{uv}) \le \nu_1(\mathrm{u}) \vee \nu_1(\mathrm{v})$ $\beta_2(\mathrm{uv}) \le \beta_1(\mathrm{u}) \wedge \beta_1(\mathrm{v})$

and $0 \le \mu_2(\mathrm{uv})+\nu_2(\mathrm{uv})+\beta_2(\mathrm{uv}) \le 1$ for every $\mathrm{uv} \in \mathrm{E}$.

In a hesitancy fuzzy graph $G(V,E)$ there is a strong edge between every pair of vertices, then $G(V,E)$ is said to be as Complete hesitancy fuzzy graph.

The cardinality of the vertex $v \in V$ in the hesitancy fuzzy graph $G(\mathrm{V,E})$ is defined by $|v| = \left[\dfrac{1+\mu_1(v)+\beta_1(v)-\gamma_1(v)}{3}\right]$.

The neighborhood degree and effective neighborhood degree of the vertex $u \in V$ in the hesitancy fuzzy graph $G(\mathrm{V,E})$ is defined by $d_N(u) = \sum_{v \in N(u)} |v|$ The order of the vertex $u \in V$ in the hesitancy fuzzy graph $G(\mathrm{V,E})$ is defined by $O(G) = \sum_{v \in V} |v|$.

An edge $uv \in E$ in a hesitancy fuzzy graph $G(\mathrm{V,E})$, is said to be an strong edge such that $\mu_2(\mathrm{uv}) = \mu_1(\mathrm{u}) \wedge \mu_1(\mathrm{v})$, $\gamma_2(\mathrm{uv}) = \gamma_1(\mathrm{u}) \vee \gamma_1(\mathrm{v})$, $\beta_2(\mathrm{uv}) = \beta_1(\mathrm{u}) \wedge \beta_1(\mathrm{v})$. The vertex $u \; and \; v$ are said to be adjacent vertices and neighborhood vertices. The neighborhood set $N(u)$ is set all vertices that are adjacent to the vertex u.

A set D of V is said to be dominating set of a hesitancy fuzzy graph $G(A,B)$ if every $v \in V - D$ there exits $u \in D$ such that u dominates v.A dominating set D of a hesitancy fuzzy graph $G(A,B)$ is called minimal dominating set of G, if every node $v \in D$, $D - \{v\}$ is not a dominating set. The dominating number $\gamma_{hf}(G)$ of the hesitancy fuzzy graph $G(A,B)$ is the minimum cardinality taken over all minimal dominating set of G.

REGULAR DOMINATING SET

In this section the idea of regular domination in Hesitancy fuzzy graphs and also discusses some properties and bounds of a Hesitancy regular domination number in fuzzy graphs.

Definition 3.1 A set $S \subseteq V$ is said to be a regular dominating set in Hesitancy fuzzy graphs $G(V,E)$ if

i) Every vertex $u \in V - S$ is adjacent to some vertex in S.

ii) Every vertex in $S \subseteq V$ has the same degree.

Minimum cardinality among all the regular dominating sets is called the regular domination number $\gamma_{HR}(G)$ of $G(V,E)$.

Theorem 3.1: In a regular hesitancy fuzzy graph $G(V,E)$ then every dominating set is a regular dominating set of $G(V,E)$.

Proof: Let $G(V,E)$ be a hesitancy regular fuzzy graph. Therefore degree of every vertex in $G(V,E)$ are unique. This implies every dominating set is a regular dominating set of $G(V,E)$.

Definition 3.2. Let $G_1(V_1,E_1)$ and $G_2(V_2,E_2)$ are two hesitancy fuzzy graphs. The union of G_1 and G_2 is defined by

$$(\mu_{11} \cup \mu_{12})(u) = \begin{cases} \mu_{11}(u) & if \; u \in V_1 \\ \mu_{12}(u) & if \; u \in V_2 \end{cases}, \qquad (\upsilon_{11} \cup \upsilon_{12})(u) = \begin{cases} \upsilon_{11}(u) & if \; u \in V_1 \\ \upsilon_{12}(u) & if \; u \in V_2 \end{cases}$$

$$(\beta_{11} \cup \beta_{12})(u) = \begin{cases} \beta_{11}(u) & if \; u \in V_1 \\ \beta_{12}(u) & if \; u \in V_2 \end{cases}$$ and the edges of the form

$$(\mu_{12} \cup \mu_{22})(uv) = \begin{cases} \mu_{12}(uv) & , \; if \; uv \in E_1 \\ \mu_{22}(uv) & , \; if \; uv \in E_2 \end{cases} \quad (\upsilon_{12} \cup \upsilon_{22})(uv) = \begin{cases} \upsilon_{12}(uv) & , if \; uv \in E_1 \\ \upsilon_{22}(uv) & , if \; uv \in E_2 \end{cases}$$

$$(\beta_{12} \cup \beta_{22})(uv) = \begin{cases} \beta_{12}(uv) & , if \ uv \in E_1 \\ \beta_{22}(uv) & , if \ uv \in E_2 \end{cases}$$

Theorem 3.2: Let $G_1(V_1, E_1)$ and $G_2(V_2, E_2)$ are two HFG. Let D_1 *and* D_2 be the minimal K-regular dominating sets of $G_1(V_1, E_1)$ and $G_2(V_2, E_2)$ respectively. Then the regular dominating number of $G_1 \cup G_2$ is $\gamma_{HR}(G_1 \cup G_2) = |D_1| + |D_2|$.

Proof: Let $G_1(V_1, E_1)$ and $G_2(V_2, E_2)$ are two HFG. Assume D_1 *and* D_2 be the minimal regular dominating sets of $G_1(V_1, E_1)$ and $G_2(V_2, E_2)$ respectively. If every vertex $u \in G_1 \cup G_2$ this implies $u \in G_1$ or $u \in G_2$ therefore there is a vertex $v \in D_1$ or $v \in D_2$ such that 'v' regularly dominates $u \in G_1 \cup G_2$. Since D_1 *and* D_2 be the regular dominating sets of $G_1(V_1, E_1)$ and $G_2(V_2, E_2)$ respectively. The regular dominating number of $G_1 \cup G_2$ is $\gamma_{HR}(G_1 \cup G_2) = |D_1| + |D_2|$. Hence proved.

Example 3.1

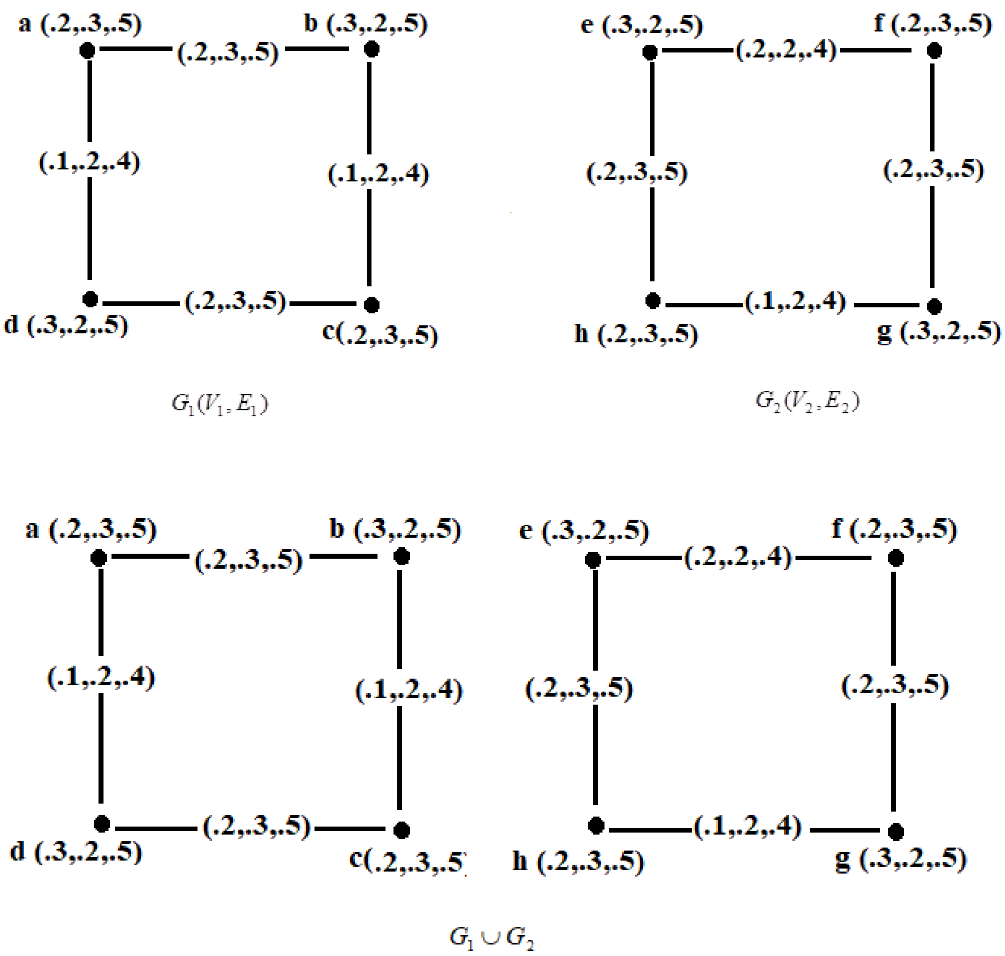

Figure 3.1

In the figure 3.1, the degree of the vertices in $G_1(V_1, E_1)$ and $G_2(V_2, E_2)$ are $d(a) = 0.53$, $d(b) = 0.47$ $d(c) = 0.47$, $d(d) = 0.53$, and $d(e) = 0.47$, $d(f) = 0.53$, $d(g) = 0.47$,

$d(h) = 0.53$. The regular dominating set of $G_1(V_1, E_1)$ and $G_2(V_2, E_2)$ are $D_1 = \{a, c\}$ and $D_2 = \{f, h\}$.. The regular dominating set of $(G_1 \cup G_2)$ is $D = \{a, c, f, h\}$ and the minimal dominating number of the graph $(G_1 \cup G_2)$ is $\gamma_{RF}(G_1 \cup G_2) = 1.88$.

Definition 3.3: Let $G_1(V_1, E_1)$ and $G_2(V_2, E_2)$ are two hesitancy fuzzy graphs. The join of G_1 and G_2 is defined by

$$(\mu_{11} + \mu_{12})(u) = \begin{cases} \mu_{11}(u) & if\ u \in V_1 \\ \mu_{12}(u) & if\ u \in V_2 \end{cases}, \quad (\upsilon_{11} + \upsilon_{12})(u) = \begin{cases} \upsilon_{11}(u) & if\ u \in V_1 \\ \upsilon_{12}(u) & if\ u \in V_2 \end{cases}$$

$$(\beta_{11} + \beta_{12})(u) = \begin{cases} \beta_{11}(u) & if\ u \in V_1 \\ \beta_{12}(u) & if\ u \in V_2 \end{cases}$$ and Edge set E is defined by

$$(\mu_{12} + \mu_{22})(uv) = \begin{cases} \mu_{12}(uv)\ , & if\ uv \in E_1 \\ \mu_{22}(uv)\ , & if\ uv \in E_2 \\ \mu_{11}(u) \wedge \mu_{21}(v) & otherwise \end{cases} \quad (\upsilon_{12} + \upsilon_{22})(uv) = \begin{cases} \upsilon_{12}(uv)\ , & if\ uv \in E_1 \\ \upsilon_{22}(uv)\ , & if\ uv \in E_2 \\ \upsilon_{11}(u) \vee \upsilon_{21}(v) & otherwise \end{cases}$$

$$(\beta_{12} + \beta_{22})(uv) = \begin{cases} \beta_{12}(uv)\ , & if\ uv \in E_1 \\ \beta_{22}(uv)\ , & if\ uv \in E_2 \\ \beta_{11}(u) \wedge \beta_{21}(v) & otherwise \end{cases}$$

Theorem 3.3: The sets D_1, D_2 be a K-regular dominating set of the hesitancy fuzzy graphs $G_1(V_1, E_1)$ and $G_2(V_2, E_2)$ respectively. If order of $G_1(V_1, E_1)$ and $G_2(V_2, E_2)$ are unique, then $\gamma_{HR}(G_1 + G_2) = \min\{|D_1|, |D_2|\}$.

Proof: Let $G_1(V_1, E_1)$ and $G_2(V_2, E_2)$ be hesitancy fuzzy graphs and The sets D_1, D_2 be regular dominating sets of the hesitancy fuzzy graphs $G_1(V_1, E_1)$ and $G_2(V_2, E_2)$ respectively. . If order of $G_1(V_1, E_1)$ and $G_2(V_2, E_2)$. This implies we get $O(G_1) = O(G_2)$. In $G_1 + G_2$ every vertex in $G_1(V_1, E_1)$ is adjacent to every vertices in $G_2(V_2, E_2)$ and vise-versa. Thisimplies the sets D_1, D_2 are dominating sets of $G_1 + G_2$ and the degree of the vertices in $G_1 + G_2$ are

$$d(v) = \begin{cases} (d_{G_1}(v) + O(G_2)), if\ v \in G_1 \\ (d_{G_2}(v) + O(G_1)), if\ v \in G_2 \end{cases}$$

Thisimplies $d(u) = d(v)$, $\forall u, v \in D_1\ or\ u, v \in D_2$ in $G_1 + G_2$. Hence $D_1\ or\ D_2$ be a regular dominating sets of $G_1 + G_2$. Therefore the minimal dominating number of $G_1 + G_2$ is

$$\gamma_R(G_1 + G_2) = \min\{|D_1|, |D_2|\}$$

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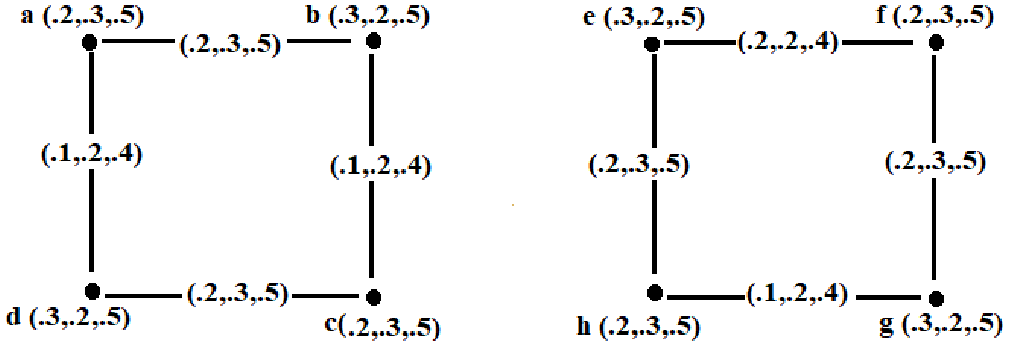
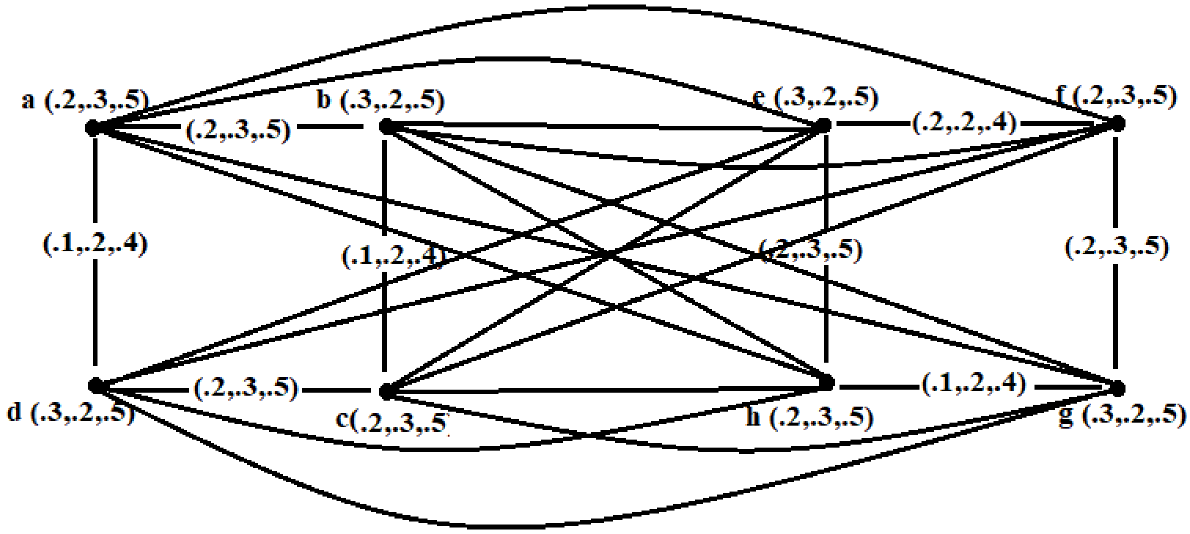
Example 3.2:



$G_1(V_1, E_1)$ $G_2(V_2, E_2)$



$G_1 + G_2$

Figure 3.3

Edge	Value	Edge	Value	Edge	Value	Edge	Value
Ae	(.2,.3,.5)	be	(.3,.2,.5)	ce	(.2,.3,.5)	de	(.3,.2,.5)
Af	(.2,.3,.5)	bf	(.2,.3,.5)	cf	(.2,.3,.5)	df	(.2,.3,.5)
Ag	(.2,.3,.5)	bg	(.3,.2,.5)	cg	(.2,.3,.5)	dg	(.3,.2,.5)
Ah	(.2,.3,.5)	bh	(.2,.3,.5)	ch	(.2,.3,.5)	dh	(.2,.3,.5)

In the figure 3.2, the degree of the vertices in $G_1(V_1, E_1)$ and $G_2(V_2, E_2)$ are $d(a) = 0.2$, $d(b) = 0.3$ $d(c) = 0.4$, $d(d) = 0.3$, and $d(e) = 0.3$, $d(f) = 0.4$, $d(g) = 0.3$, $d(h) = 0.5$. The regular dominating set of $G_1(V_1, E_1)$ and $G_2(V_2, E_2)$ are $D_1 = \{b, d\}$ and $D_2 = \{e, g\}$ .. The degree of the vertices in $(G_1 + G_2)$ are $d(a) = 1.7$, $d(b) = 1.8$, $d(c) = 1.9$, $d(d) = 1.8$, $d(e) = 1.5$,

$d(f)=1.6, d(g)=1.5, d(h)=1.7$. The graph (G_1+G_2) does not contain a regular dominating set since $O(G_1)\neq O(G_2)$

Conclusion:

In future we define variousdomination set and various domination number in hesitancy fuzzy graph and investigate some properties and bounds of regular domination number in varioushesitancy fuzzy graphs.

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