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On Nano α*-continuous Functions in Nano Topological Spaces

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ABSTRACT

This paper focuses on Nano α*-continuous functions (Nα*-continuous functions) in nano topological spaces and certain properties are investigated. We also investigate the concept of contra Nα*-continuous functions and discussed their relationships with other forms of nano continuous functions. Further, we have given an appropriate examples to understand the abstract concepts clearly.

Keywords: Nano continuous function, contra nano continuous function, Nano α*-continuous, contra Nα*-continuous functions

1. Introduction

Topology is a branch of Mathematics through which we elucidate and investigate the ideas of continuity , within the framework of Mathematics. The study of topological spaces, their continuous mappings and general properties make up one branch of topologies known as general topology. In 197', Levine [1'] introduced the concept of generalized closed sets in topological spaces. This concept was found to be useful to develop many results in general topology. In 1991, Balachandran et.al [1] introduced and investigated the notion of generalized continuous functions in topological spaces. In 2''8, Jafari et.al [6] introduced g-closed sets in topological spaces. The notion of nano topology was introduced by Lellis Thivagar [8] which was defined in terms of approximations and boundary region of a subset of an universe using an equivalence relation on it. He also established and analyzed the nano forms of weakly open sets such as nano α-open sets, nano semi-open sets and nano pre-open sets. Bhuvaneswari and Mythili Gnanapriya [4], introduced and studied the concept of Nano generalized-closed sets in nano topological spaces. In 2'17, Lalitha [7] defined the concept of Ng closed and open sets in nano topological spaces.

2. Preliminaries

Definition 2.1. [9] Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as indiscernibility relation. Then U is divided into disjoint equivalence classes. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U,R) is said to be the approximation space. Let $X \subseteq U$. Then,

(i) The lower approximation of x with respect to R is the set of all objects, which can be for certain classified as X with respect to R and is denoted by $L_R(X)$. $L_R(X) = \cup_{x\in U}\{R(x) : R(x) \subseteq X\}$ where R(x) denotes the equivalence class determined by $x \in U$.

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(ii) The upper approximation of x with respect to R is the set of all objects which can be possibly classified as X with respect to R and is denoted by $U_R(X)$. $U_R(X) = \cup_{x\in U}\{R(x) : R(x)\cap X \neq \emptyset\}$

(iii) The boundary region of x with respect to R is the set of all objects which can be classified neither as X nor as not-X with respect to R and it is denoted by $B_R(X)$. $B_R(X) = U_R(X) - L_R(X)$.

Definition 2.2. [8] Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{U,\varphi,L_R(X), U_R(X),B_R(X)\}$ where $X \subseteq U$. $\tau_R(X)$ satisfies the following axioms:

(1) U and $\varphi \in \tau_R(X)$
(2) The union of elements of any subcollection of $\tau_R(X)$ is in $\tau_R(X)$.

(3) The intersection of the elements of any finite subcollection of $\tau_R(X)$ is in $\tau_R(X)$

That is, $\tau_R(X)$ forms a topology on U is called the nano topology on U with respect to X. We call $\{U,\tau_R(X)\}$ is called the nano topological space.

Definition 2.3. [8] If $(U,\tau_R(X))$ is Nano topological space with respect to X where $X \subseteq U$ and if $A \subseteq U$,then

(1) The nano interior of the set A is defined as the union of all nano open subsets contained in A and is denoted by Nint(A). Nint(A) is the largest nano open subset of A.

(2) The nano closure of the set A is defined as the intersection of all nano closed sets containing A and is denoted by Ncl(A). Ncl(A) is the smallest nano closed set containing A.

Definition 2.4. Let $(U,\tau_R(X))$ be a nano topological space and $A \subseteq U$. Then A is said to be ,

(1) Nano semi-closed [8], if $Nint(Ncl(A)) \subseteq A$.

(2) Ng-closed [2], if $Ncl(A) \subseteq G$ whenever $A \subseteq G$ and G is nano open.

(3) Ngs-closed [3], if $Nscl(A) \subseteq G$ whenever $A \subseteq G$ and G is nano open.

(4) Ngp-closed [3], if $Npcl(A) \subseteq G$ whenever $A \subseteq G$ and G is nano open.

(5) Ngsp-closed [11], if $Nspcl(A) \subseteq G$ whenever $A \subseteq G$ and G is nano open.

(6) Ng-closed [7], if $Ncl(A) \subseteq G$ whenever $A \subseteq G$ and G is nano semi-open.

(7) Nαg closed [4], if $N\alpha cl(A) \subseteq G$ whenever $A \subseteq G$ and G is nano open.

(8) Nsg-closed [5] , if $Nscl(A) \subseteq G$, whenever $A \subseteq G$ and G is nano semi open.

Definition 2.5. Let $(U,\tau_R(X))$ and $(V,\tau_R'(Y))$ be a nano topological spaces. Then the function $f : (U,\tau_R(X)) \rightarrow (V,\tau_R'(Y))$ is nano continuous on U, if the inverse image of every nano open set in V is nano open set in U.

(1) nano semi-continuous [13] if $f^{-1}(B)$ is nano semi-open on U for every nano-open set B in V .

(2) nano pre-continuous [13] if $f^{-1}(B)$ is nano pre-open in U for every nano-open set B in V .

Definition 2.6. [12] A function $f : (U,\tau_R(X)) \rightarrow (V,\tau_R'(Y))$ is called contra nano semi pre (or contra Nβ) continuous function if the inverse image of every nano-open set of V is Nβ-closed set in U.

3. Nano α*-continuous Functions

Definition 3.1. A function $f : (U,\tau_R(X)) \rightarrow (V,\tau_R'(Y))$ is called a nano α*-continuous if $f^{-1}(O)$ is a nano α*-open set of $(U,\tau_R(X))$ for every nano open set O of $(V,\tau_R'(Y))$.

Theorem 3.2. Every nano continuous function is a nano α*-continuous.

Proof: Let $f : (U,\tau_R(X)) \rightarrow (V,\tau_R'(Y))$ be a nano continuous function. Let O be a nano open set in $(V,\tau_R'(Y))$. Since, f is nano continuous, we have then $f^{-1}(O)$ is nano open in $(U,\tau_R(X))$. Hence $f^{-1}(O)$ is nano α*open in $(U,\tau_R(X))$. Therefore, f is nano α*-continuous.

Remark 3.3. The converse of the above theorem need not be true as seen from the example.

Example 3.4. Let U = V = {a,b,c,d}, U/R = {{a, b},{c},{d}}, X = {a,c}, $\tau_R(X)$ = {V,φ,{c},{a, b, c},{a, b}}, V/R = {{a,b}, {c}, {d}},Y = {a, d},$\tau_R'(Y)$ = {U,φ,{d},{a, b, d},{a, b}.LetNα*(U,X) = {U,φ,{a},{b},{c},{d},{a, b},{a, d},{b, d},{a, b, d}} and Nα*(V,Y) = {U,φ,,{c},{d},{a, b},{b, c},{c, d},{a, b, c},{a, b, d},{a, c, d},{b, c, d}} f(a) = b, f(b) = a, f(c) = d, f(d) = c, is nano α*-continuous but not nano continuous.

Theorem 3.5. Every nano g continuous map is nano α*continuous but not conversely.

Proof: Let $f : (U,\tau_R(X)) \rightarrow (V,\tau_R'(Y))$ be a nano g continuous map and O be any nano open set in V. Since, f is nano g continuous, then $f^{-1}(O)$ is nano g-open in $(U,\tau_R(X))$. Hence, $f^{-1}(O)$ is nano α*open in $(U,\tau_R(X))$. Therefore, f is nano α*-continuous.

Example 3.6. Let U = V = {a,b,c}, U/R = {{a,b}, {c}, {d}},X = {a, c},$\tau_R(X)$ = {V,φ,{c},{a, b, c},{a, b}} and $\tau_R^c(X)$ ={{a, b, d},{d},{c, d}}, V/R = {{a,b}, {c}, {d}},Y = {a, d},$\tau_R'(Y)$ = {U,φ,{d},{a, b, d},{a, b}}and$\tau_R^c(Y)$ = {{a,b,c}, {c},{c,d}},Letf(a) = b,f(b) = a,f(c) = d,f(d) = cisnanoα*-continuous but not nano continuous.

Theorem 3.7. Every nano α-continuous map is nano α*continuous but not conversely.

Proof: Let $f : (U,\tau_R(X)) \rightarrow (V,\tau_R'(Y))$ be a nano α^*continuous map and O be any nano open set in V . Since, f is nano α continuous, then$f^{-1}(O)$ is nano α^*-open in $(U,\tau_R(X))$. Hence $f^{-1}(O)$ is nano α^*open in $(U,\tau_R(X))$. Therefore, f is nano α^*-continuous.

Example 3.8. Same as Example 3.6.

4. Contra nano α^*-continuous functions

In this section we study contra nano α^*-continuous functions and look into basic properties.

Definition 4.1. A function $f : (U,\tau_R(X)) \rightarrow (V,\tau_R'(Y))$ is called contra nano α^*continuous functions if $f^{-1}(O)$ is nano α^*closed in $(U,\tau_R(X))$ for every nano open set O in $(V,\tau_R'(Y))$.

Theorem 4.2. Every contra nano continuous function is a contra nano α^*continuous.

Proof: Let $f : (U,\tau_R(X)) \rightarrow (V,\tau_R'(Y))$ be a contra nano continuous function. Let O be a nano open set in $(V,\tau_R'(Y))$. Since, f is contra nano continuous, we have $f^{-1}(O)$ is nano closed in $(U,\tau_R(X))$.
Hence, $f^{-1}(O)$ is nano α^*closed in $(U,\tau_R(X))$. Therefore, f is contra nano α^*continuous.

Remark 4.3. The converse of the above theorem need not be true.

Example 4.4. Let U = V = {a,b,c,d}, U/R = {a,b}, {c}, {d}},X = {a, c},$\tau_R(X)$ = {V,φ,{c},{a, b, c},{a, b}} and $\tau_{R}c(X)$ = {{d},{a,b,d},{c,d}},V/R = {a, b},{c},{d}}, Y ={a, d},$\tau_R'(Y)$ = {U,φ,{d},{a, b, d},{a, b}} and $\tau_R'_c(Y)$ ={{a, b, c},{c},{c, d}}. Let f(a) = b, f(b) = a, f(c) = d, f(d) = c is contra nano α^*-continuous but not contra nano continuous.

Theorem 4.5. Every contra nano g continuous map is contra nano α^*continuous.

Proof: Let $f : (U,\tau_R(X)) \rightarrow (V,\tau_R'(Y))$ be a contra nano g continuous map and O be any nano open set in V. Since, f is contra nano g continuous, we have $f^{-1}(O)$ is nano g-closed in $(U,\tau_R(X))$.
Hence, $f^{-1}(O)$ is nano α^*closed in $(U,\tau_R(X))$. Therefore, f is contra nano α^*continuous.

Theorem 4.6. Every contra nano α continuous map is contra nano α^*continuous.

Proof: Let $f : (U,\tau_R(X)) \rightarrow (V,\tau_R'(Y))$ be a contra nano α continuous map and O be any nano open set in V. Since, f is contra nano α^*continuous, then $f^{-1}(O)$ is nano α^*-closed in $(U,\tau_R(X))$.
Hence, $f^{-1}(O)$ is nano α^*closed in $(U,\tau_R(X))$. Therefore, f is contra nano α^*continuous.

Remark 4.7. The converse of the above theorem 4.5 and 4.6 need not be true.

Example 4.8. Let U = V = {a,b,c}, U/R = {a,b}, {c}, {d}},X = {a, c},$\tau_R(X)$ = {V,φ,{c},{a, b, c},{a, b}} and $\tau_{R}c(X)$ ={{a, b, d},{d},{c, d}}, V/R = {{a,b}, {c}, {d}},Y = {a, d},$\tau_R'(Y)$ = {U,φ,{d},{a, b, d},{a, b}} and $\tau_{R}c(Y)$ = {{a,b,c}, {c}{c, d}}, Let f(a) = b, f(b) = a, f(c) = d, f(d) = c is contra nano α^*-continuous but not contra nano continuous (resp. contra nano α-continuous). **Remark 4.9.** The composition of two contra nano α^*continuous need not be contra nano α^*continuous.

Theorem 4.1'. Let $f : (U,\tau_R(X)) \rightarrow (V,\tau_R'(Y))$ be a map. The following are equivalent.

1. f is contra nano α^*continuous.
2. The inverse image of a nano closed set F of V is nano α^*open in U.

Proof: Let F be a nano closed set in V. Then V \F is an nano open set in V. By the assumption of (1), $f^{-1}(V \setminus F) = U \setminus f^{-1}(F)$ is nano α^*closed in U. It implies that $f^{-1}(F)$ is nano α^*open in U.
Converse is similar.

Theorem 4.11. The following are equivalent for a function $f : (U,\tau_R(X)) \rightarrow (V,\tau_R'(Y))$. Assume that $\alpha^*O(U,\tau_R(X))$ (resp $\alpha^*C(U,\tau_R(X))$) is nano closed under any union (resp intersection).

1. f is contra nano α^*continuous.
2. The inverse image of a nano closed set F of V is nano α^*open in U
3. For each $x \in U$ and each nano closed set B in V with $f(x) \in B$, there exists an nano α^*open set A in U such that $x \in A$ and $f(A) \subset B$
4. $f(\alpha^*cl(A)) \subset Nker(f(A))$ for every subset A of U.
5. $\alpha^*cl(f^{-1}(B)) \subset f^{-1}(NkerB)$ for every subset B of V.

Proof: (1) ⇒ (3) Let $x \in U$ and B be a nano closed set in V with $f(x) \in B$. By (1), it follows that $f^{-1}(V \setminus B) = U \setminus f^{-1}(B)$ is nano α^*closed and so $f^{-1}(B)$ is nano α^*open. Take $A = f^{-1}(B)$. We obtain that $x \in A$ and $f(A) \subset B$.
(3) ⇒ (2) Let B be a nano closed set in V with $x \in f^{-1}(B)$. Since, $f(x) \in B$, by (3), there exist a nano α^*open set A in U containing x such that $f(A) \subset B$. It follows that $x \in A \subset f^{-1}(B)$. Hence, $f^{-1}(B)$ is nano α^*open.
(2) ⇒ (1) Follows from the previous theorem
(2) ⇒ (4) Let A be any subset of U. Let $y \notin Nker(f(A))$. Then there exists a nano closed set F containing y such that $f(A) \cap F = \varphi$. Hence, we have $A \cap f^{-1}(F) = \varphi$ and $\alpha^*cl(A) \cap f^{-1}(F) = \varphi$. Hence, we obtain $f(\alpha^*cl(A)) \cap F = \varphi$ and $y \notin f(\alpha^*cl(A))$. Thus, $f(\alpha^*cl(A)) \subset ker(f(A))$.

(4) ⇒ (5) Let B be any subset of V. By (4) and lemma [2.13] $f(\alpha^*cl(f^{-1}(B))) \subset (NkerB)$ and $\alpha^*cl(f^{-1}(B)) \subset f^{-1}(NkerB)$

(5) ⇒ (1) Let B be any nano open set in V. By (5) and lemma [2.13] $\alpha^*cl(f^{-1}(B)) \subset f^{-1}(NkerB) = f^{-1}(B)$ and $\alpha^*cl(f^{-1}(B)) = f^{-1}(B)$. We obtain that $f^{-1}(B)$ is nano α^*closed in U.

Theorem 4.12. If $f : (U,\tau_R(X)) \rightarrow (V,\tau_R'(Y))$ is nano α^*irresolute and $g : (V,\tau_R'(Y)) \rightarrow (W,\tau_R''(Z))$ is contra nano α^*continuous, then their composition $g \circ f : (U,\tau_R(X)) \rightarrow (W,\tau_R''(Z))$ is contra nano α^*continuous.

Proof: Let O be any nano open set in $(W,\tau_R''(Z))$. Since, g is contra nano α^*continuous, we have$g^{-1}(O)$ is nano α^*closed in $(V,\tau_R'(Y))$ and since f is nano α^*irresolute, we have $f^{-1}(g^{-1}(O))$ is nano α^*closed in $(U,\tau_R(X))$. Therefore, $g \circ f$ is contra nano α^*continuous.

Theorem 4.13. If $f : (U,\tau_R(X)) \rightarrow (V,\tau_R'(Y))$ is contra nano α^*continuous $g : (V,\tau_R'(Y)) \rightarrow (W,\tau_R''(Z))$ is nano continuous, then their composition $g \circ f : (U,\tau_R(X)) \rightarrow (W,\tau_R''(Z))$ is contra nano α^*continuous.

Let O be any nano open set in $(W,\tau_R''(Z))$. Since, g is nano continuous, we have $g^{-1}(O)$ is nano open in $(V,\tau_R'(Y))$ and since f is contra nano α^*continuous, we have $f^{-1}(g^{-1}(O))$ is nano α^*closed in $(U,\tau_R(X))$. Therefore, $g \circ f$ is contra nano α^*continuous.

Theorem 4.14. If $f : (U,\tau_R(X)) \rightarrow (V,\tau_R'(Y))$ is contra nano α^*continuous $g : (V,\tau_R'(Y)) \rightarrow (W,\tau_R''(Z))$ is nano continuous, then their composition $g \circ f : (U,\tau_R(X)) \rightarrow (W,\tau_R''(Z))$ is contra nano α^*continuous.

Proof: Let O be any nano open set in $(W,\tau_R''(Z))$. Since, g is nano continuous, we have $g^{-1}(O)$ is nano open in $(V,\tau_R'(Y))$ and since f is contra nano α^*continuous, we have $f^{-1}(g^{-1}(O))$ is nano α^*closed in $(U,\tau_R(X))$. Since every nano α closed set is nano α^*closed. We have $f^{-1}(g^{-1}(O))$ is nano α^*closed in $(U,\tau_R(X))$. Therefore, $g \circ f$ is contra nano α^*continuous.

Theorem 4.15. If $f : (U,\tau_R(X)) \rightarrow (V,\tau_R'(Y))$ is contra nano α^*continuous $g : (V,\tau_R'(Y)) \rightarrow (W,\tau_R''(Z))$ is nano g-continuous, then their composition $g \circ f : (U,\tau_R(X)) \rightarrow (W,\tau_R''(Z))$ is contra nano α^*continuous.

Proof: Let O be any nano open set in $(W,\tau_R''(Z))$. Since, g is nano g-continuous, then g^{-1} is nano g-open in $(V,\tau_R'(Y))$ and since f is contra nano α^*continuous, then $f^{-1}(g^{-1}(O))$ is nano α^*closed in $(U,\tau_R(X))$. Therefore, $g \circ f$ is contra nano α^*continuous.

Theorem 4.16. If $f : (U,\tau_R(X)) \rightarrow (V,\tau_R'(Y))$ is strongly α^*continuous and $g : (V,\tau_R'(Y)) \rightarrow (W,\tau_R''(Z))$ is contra nano α^*continuous, then their composition $g \circ f : (U,\tau_R(X)) \rightarrow (W,\tau_R''(Z))$ is contra nano continuous.

Proof: Let O any nano open set in $(W,\tau_R''(Z))$. Since, g is contra nano α^*continuous, then g^{-1} is nano α^*closed in $(V,\tau_R'(Y))$ and since f is strongly nano α^*continuous, $f^{-1}(g^{-1}(O))$ is nano closed in $(U,\tau_R(X))$. Therefore, $g \circ f$ is contra nano continuous.

Theorem 4.17. If $f : (U,\tau_R(X)) \rightarrow (V,\tau_R'(Y))$ is perfectly nano α^*continuous, and $g : (V,\tau_R'(Y)) \rightarrow (W,\tau_R''(Z))$ is contra nano α^*continuous, then their composition $g \circ f : (U,\tau_R(X)) \rightarrow (W,\tau_R''(Z))$ is .

Proof: Let O any nano open set in $(W,\tau_R''(Z))$. Since every nano open set is nano α^*open set which implies O is nano α^*open in $(W,\tau_R''(Z))$. Since, g is contra nano α^*continuous, then g^{-1} is nano α^*closed in $(V,\tau_R'(Y))$ and since f is perfectly nano α^*continuous, then $f^{-1}(g^{-1}(O))$ is both nano open and nano closed in U, which implies $(g \circ f)^{-1}(O)$ is both nano open and nano closed in U. Therefore, $(g \circ f)$ is perfectly nano α^*continuous.

Theorem 4.18. Let $f : (U,\tau_R(X)) \rightarrow (V,\tau_R'(Y))$ is surjective nano α^*irresolute and nano α^*open and $g : (V,\tau_R'(Y)) \rightarrow (W,\tau_R''(Z))$ be any function. Then $(g \circ f) : (U,\tau_R(X)) \rightarrow (W,\tau_R''(Z))$ is contra nano α^*continuous if and only if g is contra nano α^*continuous.

The if part is easy to prove. To prove the only if part, let F be any nano closed set in $(W,\tau_R''(Z))$. Since $(g \circ f)$ is contra nano α^*continuous, then $f^{-1}(g^{-1}(F))$ is nano α^*open in $(U,\tau_R(X))$ and since f is nano α^*open surjection, then $f(f^{-1}(g^{-1}(F))) = g^{-1}(F)$ is nano α^*open in $(V,\tau_R'(Y))$. Therefore, g is contra nano α^*continuous.

Theorem 4.19. Let $f : (U,\tau_R(X)) \rightarrow (V,\tau_R'(Y))$ be a function and X be a $\alpha^*T_{1/2}$ space. Then the following are equivalent:

1. f is contra nano continuous
2. f is contra nano α^*continuous

Proof: (1) ⇒ (2) Let O be any nano open set in $(V,\tau_R'(Y))$. Since f is contra nano continuous, $f^{-1}(O)$ is nano closed in $(U,\tau_R(X))$ and since Every nano closed set is nano α^*closed, $f^{-1}(O)$ is nano α^*closed in $(U,\tau_R(X))$. Therefore, f is contra nano α^*continuous.

(2) ⇒ (1) Let O be any nano open set in $(V,\tau_R'(Y))$. Since, f is contra nano α*continuous, $f^{-1}(O)$ is nano α*closed in $(U,\tau_R(X))$ and since U is nano α*$T_{1/2}$ space, $f^{-1}(O)$ is nano closed in $(U,\tau_R(X))$. Therefore, f is contra nano continuous.

Theorem 4.2'. If f : $(U,\tau_R(X)) \to (V,\tau_R'(Y))$ is contra nano α*continuous and $(V,\tau_R'(Y))$ is regular, then f is nano α*continuous.

Proof: Let x be an arbitrary point of U and O be any nano open set of V containing f(x). Since V is regular, there exists a nano open set U in V containing f(x) such that Ncl(U) ⊂O. Since, f is contra nano α*continuous, so by Theorem[3.2.12] , there exists N ∈α*O$(U,\tau_R(X))$, such that f(N) ⊂Ncl(U). Then, f(N) ⊂O. Hence, f is nano α*continuous.

Theorem 4.21. If f is nano α*continuous and if V is locally indiscrete, then f is contra nano α* continuous.

Proof: Let O be a nano open set of V. Since V is locally discrete, O is nano closed. Since, f is nano α*continuous, $f^{-1}(O)$ is nano α*closed in U. Therefore, f is contra nano α*continuous.

Theorem 4.22. If a function f : $(U,\tau_R(X)) \to (V,\tau_R'(Y))$ is nano continuous and U is a locally indiscrete space, then f is contra α*continuous.

Proof: Let O be any nano open set in (U,σ). Since f is nano continuous $f^{-1}(O)$ is nano open in U. and since U is locally discrete, $f^{-1}(O)$ is nano closed in U. Every nano closed set is nano α*closed. $f^{-1}(O)$ is nano α*closed in U. Therefore, f is contra nano α*continuous

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