International Journal of Modern Agriculture, Volume 10, No.1, 2021
ISSN: 2305-7246

INTUITIONISTIC ANTI L - FUZZY HX SEMIRING

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***ABSTRACT* : In this paper, we define the notion of intuitionistic anti L- fuzzy HX semiring of a HX ring and some of their related properties are investigated. We define the necessity and possibility operators of an intuitionistic anti L-fuzzy subset of an intuitionistic anti L- fuzzy HX semiring and discuss some of its properties.**

Keywords:* *intuitionistic fuzzy set, fuzzy HX ring, intuitionistic L- fuzzy HX semiring, intuitionistic anti L- fuzzy HX semiring, anti product in intuitionistic anti L- fuzzy HX semiring.

INTRODUCTION

In 1965, Zadeh [8] introduced the concept of fuzzy subset μ of a set X as a function from X into [0, 1] and studied their properties. With the successful upgrade of algebraic structure of group many researchers considered the algebraic structure of some other algebraic systems in which ring was considered as first. In 1988, Professor Li Hong Xing [4] proposed the concept of HX ring and derived some of its properties, then Professor Zhong [2,3] gave the structures of HX ring on a class of ring. R.Muthuraj et.al [7]., introduced the concept of fuzzy HX ring. In this paper we define a new algebraic structure of an intuitionistic anti L-fuzzy HX semiring of a HX ring and investigate some related properties. We define the necessity and possibility operators of an intuitionistic fuzzy subset of an intuitionistic anti L-fuzzy HX semiring and discuss some of its properties.

Preliminary

In this section, we site the fundamental definitions that will be used in the sequel. Throughout this paper, $R = (R ,+, \cdot)$ is a Ring, e is the additive identity element of R and xy, we mean x.y

2.1 Definition [4]

Let R be a ring. In $2^R - \{\phi\}$, a non-empty set $\vartheta \subset 2^R - \{\phi\}$ with two binary operation '+' and '.' is said to be a HX ring on R if ϑ is a ring with respect to the algebraic operation defined by

i. $A + B = \{a + b / a \in A \text{ and } b \in B\}$, which its null element is denoted by Q , and the negative element of A is denoted by – A.
ii. $AB = \{ab / a \in A \text{ and } b \in B\}$,
iii. $A (B + C) = AB + AC$ and $(B + C) A = BA + CA$.

3. Intuitionistic anti L-fuzzy HX semiring of a HX ring

In this section we define the concept of an intuitionistic anti fuzzy HX semiring of a HX ring and discuss some related results.

3.1 Definition

Let R be a ring. Let H = { $\langle x, \mu(x), \eta(x)\rangle$ / $x\in R$ } be an intuitionistic L-fuzzy set defined on a ring R, where $\mu : R\to[0,1]$, $\eta : R\to[0,1]$ such that $0 \le \mu(x) + \eta(x) \le 1$. Let $\Re \subset 2^R - \{\phi\}$ be a HX ring. An intuitionistic L-fuzzy subset $\lambda_H = \{ \langle A, \lambda_\mu(A), \lambda_\eta (A)\rangle / A\in\Re$ and $0\le \lambda_\mu(A) + \lambda_\eta (A) \le 1\}$ of $\Re$ is called an intuitionistic L-fuzzy HX semiring of $\Re$ or an intuitionistic L-fuzzy semiring induced by H if the following conditions are satisfied.

For all $A,B \in\Re$,

i. $\lambda_\mu (A + B) \ge \lambda_\mu (A) \wedge \lambda_\mu (B)$

ii. $\lambda_\mu (AB) \ge \lambda_\mu (A) \wedge \lambda_\mu (B)$

iii. $\lambda_\eta (A + B) \le \lambda_\eta (A) \vee \lambda_\eta (B)$

iv. $\lambda_\eta (AB) \le \lambda_\eta (A) \vee \lambda_\eta (B)$.

where $\lambda_\mu (A) = \min\{ \mu(x) /$ for all $x\in A \subseteq R \}$ and $\lambda_\eta (A) = \max\{\eta(x) /$ for all $x\in A \subseteq R \}$.

3.2 Definition

Let R be a ring. Let H = { $\langle x, \mu(x), \eta(x)\rangle$ / $x\in R$ } be an intuitionistic L-fuzzy set defined on a ring R, where $\mu : R\to[0,1]$, $\eta : R\to[0,1]$ such that $0 \le \mu(x) + \eta(x) \le 1$. Let $\Re \subset 2^R - \{\phi\}$ be a HX ring. An intuitionistic L-fuzzy subset $\lambda_H = \{ \langle A, \lambda_\mu(A), \lambda_\eta (A)\rangle / A\in\Re$ and $0\le \lambda_\mu(A) + \lambda_\eta (A) \le 1\}$ of $\Re$ is called an intuitionistic anti L-fuzzy HX semiring of $\Re$ or an intuitionistic anti L-fuzzy semiring induced by H if the following conditions are satisfied. For all $A,B \in\Re$,

i. $\lambda_\mu (A + B) \le \lambda_\mu (A) \vee \lambda_\mu (B)$

ii. $\lambda_\mu (AB) \le \lambda_\mu (A) \vee \lambda_\mu (B)$

iii. $\lambda_\eta (A+ B) \ge \lambda_\eta (A) \wedge \lambda_\eta (B)$

iv. $\lambda_\eta (AB) \ge \lambda_\eta (A) \wedge \lambda_\eta (B)$.

where $\lambda_\mu (A) = \max \{ \mu(x) /$ for all $x\in A \subseteq R \}$ and $\lambda_\eta (A) = \min\{\eta(x) /$ for all $x\in A \subseteq R \}$.

3.2 Remark

For an intuitionistic anti L-fuzzy HX semiring λ_μ of a HX ring $\Re$, the following result is obvious. For all $A , B \in\Re$,

i. $\lambda_\mu (A) \ge \lambda_\mu (0)$ and $\lambda_\mu (A) = \lambda_\mu (-A)$,

ii. $\lambda_\mu (A - B) = 0$ implies that $\lambda_\mu (A) = \lambda_\mu (B)$.

iii. $\lambda_\eta (A) \le \lambda_\eta (0)$ and $\lambda_\eta (A) = \lambda_\eta (-A)$,

iv. $\lambda_\eta (A - B) = 0$ implies that $\lambda_\eta (A) = \lambda_\eta (B)$.

3.3 Theorem

Let G and H be any two intuitionistic L-fuzzy sets on R. Let γ_G and λ_H be any two intuitionistic anti L-fuzzy HX semirings of a HX ring $\Re$ then their union, $\gamma_G \cup \lambda_H$ is also an intuitionistic anti L-fuzzy HX semiring of a HX ring $\Re$.

Proof

Let G = { $\langle x, \alpha(x), \beta(x)\rangle$ / $x \in R$ } and H = { $\langle x, \mu(x), \eta(x)\rangle$ / $x \in R$ } be any two intuitionistic L-fuzzy sets defined on a ring R.

Then, $\gamma_G = \{ \langle A, \gamma\alpha(A), \gamma\beta(A)\rangle / A \in \Re \}$ and $\lambda_H = \{ \langle A, \lambda\mu(A), \lambda\eta(A)\rangle / A \in \Re \}$ be any two intuitionistic anti L-fuzzy HX semirings of a HX ring $\Re$.Then,

$$\gamma_G \cup \lambda_H = \{ \langle A, (\gamma_\alpha \cup \lambda_\mu)(A), (\gamma_\beta \cap \lambda_\eta)(A)\rangle / A \in \Re \}$$

Let A,B $\in \Re$

i.
$$\begin{aligned}(\gamma_\alpha \cup \lambda_\mu)(A+B) &= \gamma_\alpha(A+B) \vee \lambda_\mu(A+B)\\ &\leq \{\gamma_\alpha(A) \vee \gamma_\alpha(B)\} \vee \{\lambda_\mu(A) \vee \lambda_\mu(B)\}\\ &= \{\gamma_\alpha(A) \vee \lambda_\mu(A)\} \vee \{\gamma_\alpha(B) \vee \lambda_\mu(B)\}\\ &= (\gamma_\alpha \cup \lambda_\mu)(A) \vee (\gamma_\alpha \cup \lambda_\mu)(B)\\ (\gamma_\alpha \cup \lambda_\mu)(A+B) &\leq (\gamma_\alpha \cup \lambda_\mu)(A) \vee (\gamma_\alpha \cup \lambda_\mu)(B).\end{aligned}$$

ii.
$$\begin{aligned}(\gamma_\alpha \cup \lambda_\mu)(AB) &= \gamma_\alpha(AB) \vee \lambda_\mu(AB)\\ &\leq \{\gamma_\alpha(A) \vee \gamma_\alpha(B)\} \vee \{\lambda_\mu(A) \vee \lambda_\mu(B)\}\\ &= \{\gamma_\alpha(A) \vee \lambda_\mu(A)\} \vee \{\gamma_\alpha(B) \vee \lambda_\mu(B)\}\\ &= (\gamma_\alpha \cup \lambda_\mu)(A) \vee (\gamma_\alpha \cup \lambda_\mu)(B)\\ (\gamma_\alpha \cup \lambda_\mu)(AB) &\leq (\gamma_\alpha \cup \lambda_\mu)(A) \vee (\gamma_\alpha \cup \lambda_\mu)(B)\end{aligned}$$

iii.
$$\begin{aligned}(\gamma_\beta \cap \lambda_\eta)(A+B) &= \gamma_\beta(A+B) \wedge \lambda_\eta(A+B)\\ &\geq \{\gamma_\beta(A) \wedge \gamma_\beta(B)\} \wedge \{\lambda_\eta(A) \wedge \lambda_\eta(B)\}\\ &= (\gamma_\beta(A) \wedge \lambda_\eta(A)) \wedge (\gamma_\beta(B) \wedge \lambda_\eta(B))\\ &= (\gamma_\beta \cap \lambda_\eta)(A) \wedge (\gamma_\beta \cap \lambda_\eta)(B)\\ (\gamma_\beta \cap \lambda_\eta)(A+B) &\geq (\gamma_\beta \cap \lambda_\eta)(A) \wedge (\gamma_\beta \cap \lambda_\eta)(B)\end{aligned}$$

iv.
$$\begin{aligned}(\gamma_\beta \cap \lambda_\eta)(AB) &= \gamma_\beta(AB) \wedge \lambda_\eta(AB)\\ &\geq \{\gamma_\beta(A) \wedge \gamma_\beta(B)\} \wedge \{\lambda_\eta(A) \wedge \lambda_\eta(B)\}\\ &= (\gamma_\beta(A) \wedge \lambda_\eta(A)) \wedge (\gamma_\beta(B) \wedge \lambda_\eta(B))\\ &= (\gamma_\beta \cap \lambda_\eta)(A) \wedge (\gamma_\beta \cap \lambda_\eta)(B)\\ (\gamma_\beta \cap \lambda_\eta)(AB) &\geq (\gamma_\beta \cap \lambda_\eta)(A) \wedge (\gamma_\beta \cap \lambda_\eta)(B)\end{aligned}$$

Hence, $\gamma_G \cup \lambda_H$ is an intuitionistic anti L-fuzzy HX semiring of a HX ring $\Re$.

3.4 Theorem

Let G and H be any two intuitionistic L-fuzzy sets on R. Let γ_G and λ_H be any two intuitionistic anti L-fuzzy HX semirings of a HX ring $\Re$ then their intersection, $\gamma_G \cap \lambda_H$ is also an intuitionistic anti L-fuzzy HX semiring of a HX ring $\Re$.

Proof

Let G = { $\langle x, \alpha(x), \beta(x)\rangle$ / $x\in R$ } and H = { $\langle x, \mu(x), \eta(x)\rangle$ / $x\in R$ } be any two intuitionistic L-fuzzy sets defined on a ring R.

Then, γ_G = { $\langle A, \gamma_\alpha(A), \gamma_\beta(A)\rangle$ / $A\in\Re$ } and λ_H = { $\langle A, \lambda_\mu(A), \lambda_\eta(A)\rangle$ / $A\in\Re$ } be any two intuitionistic anti L-fuzzy HX semirings of a HX ring $\Re$.

$$\gamma_G \cap \lambda_H = \{ \langle A, (\gamma_\alpha \cap \lambda_\mu)(A), (\gamma_\beta \cup \lambda_\eta)(A)\rangle / A\in\Re \}$$

Let $A,B \in \Re$.

i.
$$\begin{aligned}(\gamma_\alpha \cap \lambda_\mu)(A+B) &= \gamma_\alpha(A+B) \wedge \lambda_\mu(A+B)\\
&\le \{\gamma_\alpha(A) \vee \gamma_\alpha(B)\} \wedge \{ \lambda_\mu(A) \vee \lambda_\mu(B)\}\\
&= \{\gamma_\alpha(A) \wedge (\lambda_\mu(A) \vee \lambda_\mu(B))\} \vee \{ \gamma_\alpha(B) \wedge (\lambda_\mu(A) \vee \lambda_\mu(B))\}\\
&= \{[\gamma_\alpha(A) \wedge \lambda_\mu(A)] \vee [\gamma_\alpha(A) \wedge \lambda_\mu(B)]\} \vee \{[\gamma_\alpha(B) \wedge \lambda_\mu(A)] \vee [\gamma_\alpha(B) \wedge \lambda_\mu(B)]\}\\
&\le [\gamma_\alpha(A) \wedge \lambda_\mu(A)] \vee [\gamma_\alpha(B) \wedge \lambda_\mu(B)]\\
&= (\gamma_\alpha \cap \lambda_\mu)(A) \vee (\gamma_\alpha \cap \lambda_\mu)(B)\end{aligned}$$

$$(\gamma_\alpha \cap \lambda_\mu)(A+B) \le (\gamma_\alpha \cap \lambda_\mu)(A) \vee (\gamma_\alpha \cap \lambda_\mu)(B)$$

ii.
$$\begin{aligned}(\gamma_\alpha \cap \lambda_\mu)(AB) &= \gamma_\alpha(AB) \wedge \lambda_\mu(AB)\\
&\le \{\gamma_\alpha(A) \vee \gamma_\alpha(B)\} \wedge \{ \lambda_\mu(A) \vee \lambda_\mu(B)\}\\
&= \{\gamma_\alpha(A) \wedge (\lambda_\mu(A) \vee \lambda_\mu(B))\} \vee \{ \gamma_\alpha(B) \wedge (\lambda_\mu(A) \vee \lambda_\mu(B))\}\\
&= \{[\gamma_\alpha(A) \wedge \lambda_\mu(A)] \vee [\gamma_\alpha(A) \wedge \lambda_\mu(B)]\} \vee \{[\gamma_\alpha(B) \wedge \lambda_\mu(A)] \vee [\gamma_\alpha(B) \wedge \lambda_\mu(B)]\}\\
&\le [\gamma_\alpha(A) \wedge \lambda_\mu(A)] \vee [\gamma_\alpha(B) \wedge \lambda_\mu(B)]\\
&= (\gamma_\alpha \cap \lambda_\mu)(A) \vee (\gamma_\alpha \cap \lambda_\mu)(B)\end{aligned}$$

$$(\gamma_\alpha \cap \lambda_\mu)(AB) \le (\gamma_\alpha \cap \lambda_\mu)(A) \vee (\gamma_\alpha \cap \lambda_\mu)(B)$$

iii.
$$\begin{aligned}(\gamma_\beta \cup \lambda_\eta)(A+B) &= \gamma_\beta(A+B) \vee \lambda_\eta(A+B)\\
&\ge \{\gamma_\beta(A) \wedge \gamma_\beta(B)\} \vee \{ \lambda_\eta(A) \wedge \lambda_\eta(B)\}\\
&= \{\gamma_\beta(A) \vee (\lambda_\eta(A) \wedge \lambda_\eta(B)\} \wedge \{ \gamma_\beta(B) \vee (\lambda_\eta(A) \wedge \lambda_\eta(B)\}\\
&= \{[\gamma_\beta(A) \vee \lambda_\eta(A)] \wedge [\gamma_\beta(A) \vee \lambda_\eta(B)]\} \wedge \{ [\gamma_\beta(B) \vee \lambda_\eta(A)] \wedge [\gamma_\beta(B) \vee \lambda_\eta(B)]\}\\
&\ge [\gamma_\beta(A) \vee \lambda_\eta(A)] \wedge [\gamma_\beta(B) \vee \lambda_\eta(B)]\\
&= (\gamma_\beta \cup \lambda_\eta)(A) \wedge (\gamma_\beta \cup \lambda_\eta)(B)\end{aligned}$$

$$(\gamma_\beta \cup \lambda_\eta)(A+B) \ge (\gamma_\beta \cup \lambda_\eta)(A) \wedge (\gamma_\beta \cup \lambda_\eta)(B)$$

iv.
$$\begin{aligned}(\gamma_\beta \cup \lambda_\eta)(AB) &= \gamma_\beta(AB) \vee \lambda_\eta(AB)\\
&\ge \{\gamma_\beta(A) \wedge \gamma_\beta(B)\} \vee \{ \lambda_\eta(A) \wedge \lambda_\eta(B)\}\\
&= \{\gamma_\beta(A) \vee (\lambda_\eta(A) \wedge \lambda_\eta(B))\} \wedge \{ \gamma_\beta(B) \vee (\lambda_\eta(A) \wedge \lambda_\eta(B))\}\\
&= \{[\gamma_\beta(A) \vee \lambda_\eta(A)] \wedge [\gamma_\beta(A) \vee \lambda_\eta(B)]\} \wedge \{ [\gamma_\beta(B) \vee \lambda_\eta(A)] \wedge [\gamma_\beta(B) \vee \lambda_\eta(B)]\}\\
&\ge [\gamma_\beta(A) \vee \lambda_\eta(A)] \wedge [\gamma_\beta(B) \vee \lambda_\eta(B)]\\
&= (\gamma_\beta \cup \lambda_\eta)(A) \wedge (\gamma_\beta \cup \lambda_\eta)(B)\end{aligned}$$

$$(\gamma_\beta \cup \lambda_\eta)(AB) \ge (\gamma_\beta \cup \lambda_\eta)(A) \wedge (\gamma_\beta \cup \lambda_\eta)(B)$$

Hence, $\gamma_G \cap \lambda_H$ is an intuitionistic anti L-fuzzy HX semiring of a HX ring $\Re$.

3.5 Definition

Let G = { $\langle x, \alpha(x), \beta(x)\rangle$ / $x\in R$ } and H = { $\langle x, \mu(x), \eta(x)\rangle$ / $x\in R$ } be any two intuitionistic L-fuzzy sets defined on a ring R. Let $\Re_1 \subset 2^{R_1} - \{\phi\}$ and $\Re_2 \subset 2^{R_2} - \{\phi\}$ be any two HX rings.

Let $\gamma_G = \{ \langle A, \gamma_\alpha(A), \gamma_\beta(A)\rangle \ / \ A\in\Re \}$ and $\lambda_H = \{\langle A, \lambda_\mu(A), \lambda_\eta(A)\rangle \ /A\in\Re \}$ be any two intuitionistic L-fuzzy subsets of a HX ring $\Re$, then the anti product of γ_G and λ_H is defined as

$$(\gamma_G \times \lambda_H) = \{\langle (A,B), (\gamma_\alpha \cap \lambda_\mu)(A,B), (\gamma_\beta \cup \lambda_\eta)(A, B)\rangle /(A, B) \in \Re_1 \times \Re_2\},$$

where, $(\gamma_\alpha \cap \lambda_\mu)(A, B) = \gamma_\alpha(A) \wedge \lambda_\mu(B)$, for all $(A, B) \in \Re_1 \times \Re_2$,

$(\gamma_\beta \cup \lambda_\eta)(A, B) = \gamma_\beta(A) \vee \lambda_\eta(B)$, for all $(A, B) \in \Re_1 \times \Re_2$.

3.6 Theorem

Let G and H be any two intuitionistic L-fuzzy sets of R_1 and R_2 respectively. Let $\Re_1 \subset 2^{R_1} - \{\phi\}$ and $\Re_2 \subset 2^{R_2} - \{\phi\}$ be any two HX rings. If γ^G and λ^H are any two intuitionistic anti L-fuzzy HX semirings of $\Re_1$ and $\Re_2$ respectively then, $\gamma^G \times \lambda^H$ is also an intuitionistic anti L-fuzzy HX semiring of a HX ring $\Re_1 \times \Re_2$.

Proof

Let $G = \{ \langle x, \alpha(x), \beta(x)\rangle \ / \ x\in R \}$ and $H = \{ \langle x, \mu(x), \eta(x)\rangle \ / \ x\in R \}$ be any two intuitionistic L-fuzzy sets defined on a ring R.

Then, $\gamma_G = \{ \langle A, \gamma_\alpha(A), \gamma_\beta(A)\rangle \ / \ A\in\Re \}$ and $\lambda_H = \{ \langle A, \lambda_\mu(A), \lambda_\eta(A)\rangle \ / \ A\in\Re \}$ be any two intuitionistic anti L-fuzzy HX semirings of a HX ring $\Re$.Then,

$$(\gamma_G \times \lambda_H) = \{\langle (A,B), (\gamma_\alpha \cap \lambda_\mu)(A,B), (\gamma_\beta\cup \lambda_\eta)(A, B)\rangle /(A, B) \in \Re_1 \times \Re_2\},$$

where, $(\gamma_\alpha \cap \lambda_\mu)(A, B) = \gamma_\alpha(A) \wedge \lambda_\mu(B)$, for all $(A, B) \in \Re_1 \times \Re_2$,

$(\gamma_\beta \cup \lambda_\eta)(A, B) = \gamma_\beta(A) \vee \lambda_\eta(B)$, for all $(A, B) \in \Re_1 \times \Re_2$.

Here C = (A,B) and D = (E,F)

i. $$\begin{aligned}(\gamma_\alpha\cap \lambda_\mu)(C + D) &= \gamma_\alpha(C+D) \wedge \lambda_\mu(C+D)\\ &\leq \{\gamma_\alpha(C) \vee \gamma_\alpha(D)\} \wedge \{ \lambda_\mu(C) \vee \lambda_\mu (D)\}\\ &= \{ \gamma_\alpha(C) \wedge (\lambda_\mu(C) \vee \lambda_\mu (D))\} \vee \{\gamma_\alpha(D) \wedge (\lambda_\mu(C) \vee\lambda_\mu (D))\}\\ &= \{(\gamma_\alpha(C) \wedge\lambda_\mu(C)) \vee (\gamma_\alpha(C) \wedge\lambda_\mu(D))\} \vee\{ (\gamma_\alpha(D) \wedge\lambda_\mu(C)) \vee (\gamma_\alpha(D) \wedge\lambda_\mu(D))\}\\ &\leq (\gamma_\alpha(C) \wedge \lambda_\mu(C)) \vee (\gamma_\alpha(D) \wedge \lambda_\mu(D))\\ &= (\gamma_\alpha\cap \lambda_\mu)(C) \vee (\gamma_\alpha\cap \lambda_\mu)(D)\end{aligned}$$

$$(\gamma_\alpha\cap \lambda_\mu)(C + D) \leq (\gamma_\alpha\cap \lambda_\mu)(C) \vee (\gamma_\alpha\cap \lambda_\mu)(D).$$

ii. $$\begin{aligned}(\gamma_\alpha\cap \lambda_\mu)(CD) &= \gamma_\alpha(CD) \wedge \lambda_\mu(CD)\\ &\leq \{\gamma_\alpha(C) \vee \gamma_\alpha(D)\} \wedge \{ \lambda_\mu(C) \vee \lambda_\mu (D)\}\\ &= \{ \gamma_\alpha(C) \wedge (\lambda_\mu(C) \vee \lambda_\mu (D))\} \vee \{\gamma_\alpha(D) \wedge (\lambda_\mu(C) \vee \lambda_\mu (D))\}\\ &= \{(\gamma_\alpha(C) \wedge\lambda_\mu(C)) \vee (\gamma_\alpha(C) \wedge \lambda_\mu(D))\} \vee \{(\gamma_\alpha(D)\wedge\lambda_\mu(C)) \vee (\gamma_\alpha(D)\wedge\lambda_\mu(D))\}\\ &\leq (\gamma_\alpha(C) \wedge \lambda_\mu(C)) \vee (\gamma_\alpha(D) \wedge \lambda_\mu(D))\\ &= (\gamma_\alpha\cap \lambda_\mu)(C) \vee (\gamma_\alpha\cap \lambda_\mu)(D)\end{aligned}$$

$$(\gamma_\alpha\cap \lambda_\mu)(CD) \leq (\gamma_\alpha\cap \lambda_\mu)(C) \vee (\gamma_\alpha\cap \lambda_\mu)(D)$$

iii. $$\begin{aligned}(\gamma_\beta\cup \lambda_\eta)(C+D) &= \gamma_\beta(C+D) \vee \lambda_\eta(C+D)\\ &\geq \{\gamma_\beta(C) \wedge \gamma_\beta(D)\} \vee \{ \lambda_\eta(C) \wedge \lambda_\eta (D)\}\\ &= \{\gamma_\beta(C) \vee (\lambda_\eta(C) \wedge \lambda_\eta (D))\} \wedge \{\gamma_\beta(D) \vee (\lambda_\eta(C) \wedge \lambda_\eta (D))\}\\ &= \{[\gamma_\beta(C) \vee \lambda_\eta(C)] \wedge [\gamma_\beta(C) \vee \lambda_\eta(D)]\} \wedge \{[\gamma_\beta(D) \vee \lambda_\eta(C)] \wedge [\gamma_\beta(D) \vee \lambda_\eta(D)]\}\\ &\geq (\gamma_\beta(C) \vee \lambda_\eta(C)) \wedge (\gamma_\beta(D) \vee \lambda_\eta(D))\\ &= (\gamma_\beta\cup \lambda_\eta)(C) \wedge (\gamma_\beta\cup \lambda_\eta)(D)\end{aligned}$$

$(\gamma_\beta \cup \lambda_\eta)(C+D) \geq (\gamma_\beta \cup \lambda_\eta)(C) \wedge (\gamma_\beta \cup \lambda_\eta)(D)$

iv. $(\gamma_\beta \cup \lambda_\eta)(CD) = \gamma_\beta(CD) \wedge \lambda_\eta(CD)$

$\geq \{\gamma_\beta(C) \wedge \gamma_\beta(D)\} \vee \{\lambda_\eta(C) \wedge \lambda_\eta(D)\}$

$= \{\gamma_\beta(C) \vee (\lambda_\eta(C) \wedge \lambda_\eta(D))\} \wedge \{\gamma_\beta(D) \vee (\lambda_\eta(C) \wedge \lambda_\eta(D))\}$

$= \{(\gamma_\beta(C) \vee \lambda_\eta(C)) \wedge (\gamma_\beta(C) \vee \lambda_\eta(D))\} \wedge \{(\gamma_\beta(D) \vee \lambda_\eta(C)) \wedge (\gamma_\beta(D) \vee \lambda_\eta(D))\}$

$\geq (\gamma_\beta(C) \vee \lambda_\eta(C)) \wedge (\gamma_\beta(D) \vee \lambda_\eta(D))$

$= (\gamma_\beta \cup \lambda_\eta)(C) \wedge (\gamma_\beta \cup \lambda_\eta)(D)$

$(\gamma_\beta \cup \lambda_\eta)(CD) \geq (\gamma_\beta \cup \lambda_\eta)(C) \wedge (\gamma_\beta \cup \lambda_\eta)(D)$

Hence, $\gamma_G \times \lambda_H$ is an intuitionistic anti L-fuzzy HX semiring of a HX ring $\mathfrak{R}$.

3.7 Definition

Let H be an intuitionistic L-fuzzy set of R. Let $\mathfrak{R} \subset 2^R - \{\phi\}$ be a HX ring. Let λ_H be an intuitionistic L-fuzzy set of $\mathfrak{R}$. We define the following "necessity" and possibility" operations:

$\square\, \lambda_H = \{ \langle A, \lambda_\mu(A), 1-\lambda_\mu(A) \rangle \ / \ A \in \mathfrak{R}\}$

$\Diamond\, \lambda_H = \{ \langle A, 1-\lambda_\eta(A), \lambda_\eta(A) \rangle \ / \ A \in \mathfrak{R}\}$.

3.8 Theorem

Let H be an intuitionistic L-fuzzy set on R. Let λ_H be an intuitionistic anti L-fuzzy HX semiring of a HX ring $\mathfrak{R}$ then $\square\, \lambda_H$ is an intuitionistic anti L-fuzzy HX semiring of a HX ring $\mathfrak{R}$.

Proof

Let λ_H be an intuitionistic anti L-fuzzy HX semiring of a HX ring $\mathfrak{R}$. Then,

i. $\lambda_\mu(A + B) \leq \lambda_\mu(A) \vee \lambda_\mu(B)$

ii. $\lambda_\mu(AB) \leq \lambda_\mu(A) \vee \lambda_\mu(B)$

iii. $\lambda_\eta(A+B) \geq \lambda_\eta(A) \wedge \lambda_\eta(B)$

iv. $\lambda_\eta(AB) \geq \lambda_\eta(A) \wedge \lambda_\eta(B)$.

Now, $\lambda_\mu(A + B) \leq \lambda_\mu(A) \vee \lambda_\mu(B)$

$1-\lambda_\mu(A + B) \geq 1 - (\lambda_\mu(A) \vee \lambda_\mu(B))$

$\geq (1 - \lambda_\mu(A)) \wedge (1 - \lambda_\mu(B))$

That is, $1-\lambda_\mu(A + B) \geq (1 - \lambda_\mu(A)) \wedge (1 - \lambda_\mu(B))$

We have, $\lambda_\mu(AB) \leq \lambda_\mu(A) \vee \lambda_\mu(B)$

$1-\lambda_\mu(AB) \geq 1 - (\lambda_\mu(A) \vee \lambda_\mu(B))$

$\geq (1 - \lambda_\mu(A)) \wedge (1 - \lambda_\mu(B))$

That is, $1-\lambda_\mu(AB) \geq (1 - \lambda_\mu(A)) \wedge (1 - \lambda_\mu(B))$

Hence, $\square\, \lambda_H$ is an intuitionistic anti-fuzzy HX semiring of a HX ring $\mathfrak{R}$.

3.9 Theorem

Let H be an intuitionistic L-fuzzy set on R. Let λ_H be an intuitionistic anti L-fuzzy HX semiring of a HX ring $\mathfrak{R}$ then $\Diamond\lambda_H$ is an intuitionistic anti L-fuzzy HX semiring of a HX ring $\mathfrak{R}$.

Proof

Let λ_H be an intuitionistic anti L-fuzzy HX semiring of a HX ring $\mathfrak{R}$. Then,

i. $\lambda_\mu(A + B) \leq \lambda_\mu(A) \vee \lambda_\mu(B)$

ii. $\lambda_\mu(AB) \leq \lambda_\mu(A) \vee \lambda_\mu(B)$

iii. $\lambda_\eta(A+B) \geq \lambda_\eta(A) \wedge \lambda_\eta(B)$

iv. $\lambda_\eta(AB) \geq \lambda_\eta(A) \wedge \lambda_\eta(B)$.

.Now, $\lambda_\eta(A + B) \geq \lambda_\eta(A) \wedge \lambda_\eta(B)$

$$1-\lambda_\eta(A + B) \leq 1 - (\lambda_\eta(A) \wedge \lambda_\eta(B))$$
$$\leq (1 - \lambda_\eta(A)) \vee (1 - \lambda_\eta(B))$$

That is, $1-\lambda_\eta(A + B) \leq (1 - \lambda_\eta(A)) \vee (1 - \lambda_\eta(B))$

We have, $\lambda_\eta(AB) \geq \lambda_\eta(A) \wedge \lambda_\eta(B)$

$$1-\lambda_\eta(AB) \leq 1 - (\lambda_\eta(A) \wedge \lambda_\eta(B))$$
$$\leq (1 - \lambda_\eta(A)) \vee (1 - \lambda_\eta(B))$$

That is, $1-\lambda_\eta(AB) \leq (1 - \lambda_\eta(A)) \vee (1 - \lambda_\eta(B))$

Hence, $\Diamond\, \lambda_H$ is an intuitionistic anti L-fuzzy HX semiring of a HX ring $\mathfrak{R}$.

Conclusion

In this paper we introduce the concept of intuitionistic anti L-fuzzy HX semiring and discuss the basic results on HX ring. Further investigation may be in intuitionistic anti L-fuzzy HX ideals on HX ring which will give a new horizon in the further study.

Acknowledgements

The authors would like to express their sincere thanks to all our friends for their help to make this paper a successful one.

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