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Optimization Of Substitutable Perishable Inventory System With Partial Backlogging In Supply Chain

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Abstract

This article deals with the continuous review perishable inventory systems with two different substitutable items in stock. The demand for the products are independent of each other and follows an Poisson distribution with parameters $\square_1$ *and* $\square_2$ *respectively for product A and B with partial backlogging. The demands, that occurred when the inventory is out of stock, are back logged. That means these demands are satisfied as soon as the replenishment is received. The common demand for both products at distributor follows Poisson distribution with rate* $\square_d$. *The* (s_i, S_i) *operating policy is used at the lower echelon for the products. That is whenever the inventory level drops to '*s_i*' on order for* Q_i = $(S_i - s_i)$ *items is placed, the ordered items are received after a random time which is distributed as exponential. The items are perishable in nature and the life time of items of each commodity is assumed to be exponentially distributed. The retailer replenishes the stock of products from the supplier which adopts (0,* M_i*) policy. The joint probability distribution of the inventory levels of the products, at retailer and the products at supplier are obtained in the steady state case. We derive the system performance measures and calculate the long run total expected inventory cost using several instances of numerical examples.*

Key Words: Two-echelon inventory, Substitutable Product, Direct demand, Markov process.

1. Introduction

In last two decades, the inventory systems are considered at demanded items are directly delivered from stock (if available). Demand realizing during stock-out periods either result in lost sales or are satisfied only after the arrival of ordered items (backlogging). In the latter case, it is assumed that either all (full backlogging) or any predetermined amount of demand (partial backlogging) realizing during the stock-out period is satisfied. See Nahmias (1982), Kalpakam et al. (1990), Raafat (1991), Liu et al. (1999) and Yadavalli et al. (2004) for review the above classical process.

In this paper we consider a continuous review with two different substitutable perishable inventory systems at a service facility. For a brief review of multi - commodity inventory systems we refer the reader to Goyal et al. (1989), Kalpakam et al. (1993) and Anbazhagan et al. (2000).

For example, Parlar and Goyal (1984) modeled the two-substitutable-product problem as an extension of the single-period problem. For a single-period inventory problem, the result of classical news boy problem (Spearman and Hopp, 2001) can be utilized. Their results showed that the optimal order quantities for each product can be found by maximizing an expected profit function which is strictly concave for a wide range of parameters values.

Drezner et al. (1995) investigated an economic order quantity model with two ordered substitution products. That is, one can be used to substitute the other at a given unit cost. Three cases are studied: no substitution, full substitution and partial substitution. The author argued that the full substitution can not be optimal. Only partial or no substitution may be optimal. By comparing the optimal total cost of these three situations, the author draws the conclusion.

Ingene and Moinzadeh (1993) developed long run profit maximizing stocking and pricing policies in the face of unpredictable but "stationary" demand for a pair of related goods. They examined a profit maximizing company that distributes two related (substitutable) products.

Anbazhagan and Arivarignan [2,3] have analyzed two commodity inventory system undervarious ordering policies. Yadavalliet. al., [32] have analyzed a model with joint ordering policy and varying order quantities. Yadavalliet. al., [33] have considered a two commodity substitutable inventory system with Poisson demands and arbitrarily distributed lead time.

Anbazhagan et. al. [4] considered analysis of two commodity inventory system with compliment for bulk demand in which, one of the items for the major item, with random lead time but instantaneous replenishment for the gift item are considered. The lost sales for major item are also assumed when the items are out of stock.

From the above models, we extend perishable inventory systems with partial backlogging in to multi-echelon structure (Supply Chain). The rest of the paper is organized as follows. In section 2, we discussed model formulation along with some important notations used in the paper and the steady state analysis is done in section 3. Section 4 deals with the derivation of operating characteristics of the system and the cost analysis for the operation discussed in section 5. Section 6 provides Numerical illustration and discussion.

Notations / Variables	Used for
$[C]_{ij}$	The element of sub matrix at (i, j)th position of C
0	Zero matrix
λ_1, λ_2	Average demand rate for products A and B a retailer node
λ_d	Average demand for both products at distributor
μ	Average replacement rate for both products A and B at retailer
S_1, S_2	Maximum inventory level products A and B at retailer
$s_1,\ s_2$	Reorder level for products A and B at retailer
M	Maximum inventory level for both products A and B at distributor
H_1	Holding cost per item for product A at retailer
H_2	Holding cost per item for product B at retailer
H_d	Holding cost per item for both products A and B at distributor
O_1	In retailer, product A Ordering cost per item
O_2	In retailer, product B Ordering cost per item
O_d	Ordering cost per item for both products A and B at distributor
I_1	Mean inventory level for product A at retailer
I_2	Mean inventory level for product B at retailer
I_d	Mean inventory level for both products at distributor
R_1	Expected reorder rate for product A at retailer
R_2	Expected reorder rate for product B at retailer

R_d	Expected reorder rate for both products A and B at distributor
T_r	Shortage rate for products at retailer
$\sum_{i=Q}^{nQ} i$	$Q + 2Q + 3Q + \cdots + nQ.$

2. Model

2.1. The Problem Description

In this paper perishable inventory control system is considered and it is defined as follows. Two Substitutable finished products (A & B) are supplied from manufacturer to supplier which adopts (0, M) replenishment policy then the product is supplied to retailer who adopts $(s_i,\ S_i)$ policy. The demand at retailer node follows an independent Poisson distribution with rate $\lambda_1 + i\gamma$ for one product A and λ_2 for product B. Also the common demand for both product at distributor follows independent Poisson process with rate λ_d. When the inventory of one of theproduct reaches zero the demand for the product is substitutable with the other product with probability p and similar argument for another product with probability q so that $p + q = 1$. The demands that occur during the stock out periods are partially backlogged. That means these demands are satisfied as soon as the replenishment is received. The replacement of item in terms of product is made from supplier to retailer is administrated with exponential distribution having parameter $\mu > 0$. The maximum inventory level at retailer node for product A and BareS_i, and the recorder point is s_i and theordering quantity is $Q(= S_i - s_i)$items. The maximum inventory at supplier is$M(= nQ)$.

2.2. Notations and variables

We use the following notations and variables for analysis of the paper.

3. Analysis

Let $I_1(t)$ and $I_2(t)$ denote the on hand Inventory levels of product A and B respectively at retailer and $I_d(t)$denote the on hand inventory level of both products at supplier at time $t+$.

We define $I(t) = \{(I_1(t), I_2(t), I_d(t))\,;\ t \geq 0\}$ as Markov process with state space $E = \{(i,j,k)|\ i = 0,\dots,S_1,\ j = 0,1,2,\dots,S_2,\ k = Q, 2Q,\dots,nQ\}$. Since E is finite and all its states are irreducible and aperiodic. We know that every finite irreducible Markov chain is Ergodic. That is all the states are Ergodic. Hence the limiting distribution exists and is independent of the initial state.

The infinitesimal generator matrix of this process $C = \big(a(i,j,k\ :\ l,m,n)\big)_{(i,j,k)(l,m,n)}$ can be obtained from the following arguments.

- The arrival of a demand for Perishable product A at retailer make a state transition in the Markov process from (i,j,k) to $(i-1,j,k)$ with the intensity of transition $(\lambda_1 + i\,\gamma),\ i \neq 0$.
- The arrival of a demand for product B at retailer make a state transition in the Markov process from (i,j,k) to $(i,j-1,k)$ with the intensity of transition $(\lambda_2 + j\,\gamma),\ j \neq 0$.
- When the inventory level of Perishable product A is zero, then the arrival of a demand for product A at retailer make a state transition in the Markov process from $(0,j-1,k)$ to $(i-j,j,k)$ with the intensity of transition $(P\lambda_1 + \lambda_2 + j\gamma) > 0$.
- When the inventory level of product B is zero, then the arrival of a demand for product B at retailer make a state transition in the Markov process from $(i,0,k)$ to $(i-1,0,k)$ with the intensity of transition $(\lambda_1 + i\gamma + q\lambda_2) > 0$.
- The arrival of a direct demand for both products at distributormakes a state transition in the Markov process from (i,j,k) to $(i,j,k-Q)$ with the intensity of transition $\lambda_d > 0$.
- The replacement of inventory at retailer make a state transition in the Markov process from (i,j,k) to $(i+Q,j,k-Q)$ or (i,j,Q) to $(i,j+Q,k-Q)$ with the intensity of transition $\mu > 0$.

The infinitesimal generator C is given by $C = [A\ B\ 0\ 0\ A\ B\ \cdots\ 0\ 0\ 0\ 0\ \vdots\ \ldots\ \vdots\ 0\ 0\ 0\ B\ 0\ 0\ \cdots\ A\ B\ 0\ A\ ]$

The sub matrices A and B are given by
$[A]_{m\times n} = \{A_1\ n = m\ m = s_1+1, s_1+2, \ldots, S_1\ A_2\ n = m-1\ m = S_1, S_1-1, \ldots, 1\ A_3\ A_4\ 0\ \ n = m\ n = m\ otherwise\ \ m = s_1, s_1-1, \ldots, 1\ m = 0$

$[B]_{m\times n} = \{B_1\ m = n\ m = S_1+1, S_1-1, \ldots, 1, 0\ B_2\ m = n+Q\ m = s_1, s_1-1, \ldots, 1, 0\ 0\ otherwise$

Where,

$[A_1]_{m\times n} = \{\lambda_1 + m\gamma_1\ n = m-1\ m = S_1, S_1-1, \ldots, 1\ -(\lambda_1 + m\gamma_1 + \lambda_2) + n\gamma_2 + \lambda_d\ n = m\ m = s_1+1, s_1+2, \ldots, S_1\ -(\lambda_1 + m\gamma_1 + \lambda_2) + n\gamma_2 + \lambda_d + \mu\ -(P(\lambda_1 + m\gamma_1) + \lambda_2 + \mu + \lambda_d)\ 0\ \ n = m\ n = m\ otherwise\ \ m = 1, 2, \ldots, s_1\ m = 0$

$[A_2]_{m\times n} = \{\lambda_2 + n\gamma_2\ n = m\ m = S_1, S_1-1, \ldots, s_1\ P(\lambda_1 + m\gamma_1 + \lambda_2 + n\gamma_2)\ n = m\ m = 0\ 0\ otherwise$

$[A_3]_{m\times n} = \{\lambda_1 + m\gamma_1\ n = m\ m = S_1, S_1-1, \ldots, s_1\ -(\lambda_1 + m\gamma_1 + \lambda_2 + n\gamma_2 + +\lambda_d)\ n = m\ m = s_1+1, s_1+2, \ldots, S_1\ -(\lambda_1 + m\gamma_1 + \lambda_2 + n\gamma_2 + 2\mu\ + \lambda_d)\ -(P\lambda_1 + m\gamma_1 + \lambda_2 + n\gamma_2 + 2\mu + \lambda_d)\ 0\ \ n = m\ n = m\ otherwise\ \ m = 1, 2, \ldots, s_1\ m = 0$

$[A_4]_{m\times n} = \{\lambda_1 + m\gamma_1\ n = m-1\ m = S_1, S_1-1, \ldots, 1\ \ -(\lambda_1 + m\gamma_1 + q\lambda_2 + m\gamma_2 + \mu)\ n = m\ m = s_1+1, s_1+2, \ldots, S_1\ -(\lambda_1 + m\gamma_1 + q\lambda_2 + m\gamma_2 + 2\mu)\ -2\mu\ 0\ \ n = m\ n = m\ otherwise\ \ m = 1, 2, \ldots, s_1\ m = 0$

$$[B_1]_{m\times n} = \{\lambda_d\ m = n\ m = S_1, S_1-1, \ldots, 0\ 0\ otherwise$$

And

$$[B_2]_{m\times n} = \{\mu\ m = n+Q\ m = S_1, S_1-1, \ldots, 0\ 0\ otherwise$$

3.1. Steady State Analysis

The structure of the infinitesimal matrix C, reveals that the state space E of the Markov process $\{I(t): t\ \geq\ 0\}$ is finite and irreducible. Let the limiting probability distribution of the inventory level process be$\Pi_{i,j}^{k} = Pr\{(I_1(t), I_2(t), I_d(t) = (i, j, k))\}$ where $\Pi_{i,j}^{k}$ the steady state probability that the system be in state (i, j, k).

Let $\Pi_{i,j}^{k} = \{\Pi_{i,j}^{nQ}, \Pi_{i,j}^{(n-1)Q}, \ldots, \Pi_{i,j}^{Q}\}$denote the steady state probability distribution. For each $(i, j, k), \Pi_{i,j}^{k}$be obtained by solving the matrix equation denote the $\prod\quad C = 0$.

By solving the above system of equations, together with normalizing condition $\sum_{i,j,k}\quad \Pi_{i,j}^{k} = 1$ the steady probabilities of all the system states are obtained.

4. Operating characteristics

In this section we derive some important system performance measures.

4.1. Average inventory Level

The event I_1, I_2denotes the average inventory level for the products A and B respectively at retailer node and I_d denote the average inventory level at distributor node.

(i) $$I_1 = \sum_{k=Q}^{nQ}\sum_{j=0}^{S_2}\sum_{i=0}^{S_1} i.\Pi_{i,j}^{k}$$

(ii) $$I_2 = \sum_{k=Q}^{nQ} \sum_{j=0}^{S_2} \sum_{i=0}^{S_1} j.\Pi_{i,j}^{k}$$

(iii) $$I_d = \sum_{i=0}^{S_1} \sum_{j=0}^{S_2} \sum_{k=Q}^{nQ} k.\Pi_{i,j}^{k}$$

4.2. Mean Reorder Rate

Let R_1, R_2denote the mean reorder rate for perishable products A and B respectively, at retailer and R_d denote themean reorder rate for products at distributor,

(i) $$R_1 = \left(\lambda_1 + (s_1+1)\gamma_1\right) \sum_{k=Q}^{nQ} \sum_{j=0}^{S_2} \Pi_{s_1+1,j}^{k}$$

(ii) $$R_2 = \left(\lambda_2 + (s_2+1)\gamma_2\right) \sum_{k=Q}^{nQ} \sum_{i=0}^{S_1} \Pi_{i,s_2+1}^{k}$$

(iii) $$R_d = \left(\mu + \lambda_d\right) \sum_{i=0}^{S_1} \sum_{j=0}^{S_2} \Pi_{i,j}^{Q}$$

4.3. Shortage rate

Shortage occurs only at retailer for products A and B only. Let $S_r$be the shortage rate at retailer for products A and B

$$S_r = (\lambda_1 + \lambda_2) \sum_{k=Q}^{nQ} \Pi_{0,0}^{k}$$

5. Cost analysis:

In this section we impose a cost structure for the proposed model and analyze it by the criteria of minimization of long run total expected cost per unit time. The long run expected cost rate $TC(s_1, s_2, Q)$is given by

$$TC(S_1, S_2, Q) = I_1.H_1 + I_2.H_2 + I_d.H_d + R_1.O_1 + R_2.O_2 + R_d.O_d + S_r.T_r.$$

Although we have not proved analytically the convexity of the cost function $TC(s_1, s_2, Q)$ we try to do number of numerical examples to find the optimal cost.

6. Numerical illustration and Discussion

In this section we discuss the problem of minimizing the structure. We assume $H_2 \leq H_1 \leq H_d$, - i.e, the holding cost for product B at retailer node is less than that of product A and the holding cost of both products is less than that of products at distributor node. Also $O_2 \leq O_1 \leq O_d$ the ordering cost at retailer node for product B is less than that of product A. Also the ordering cost at the distributor is greater than that of both products at retailer node.

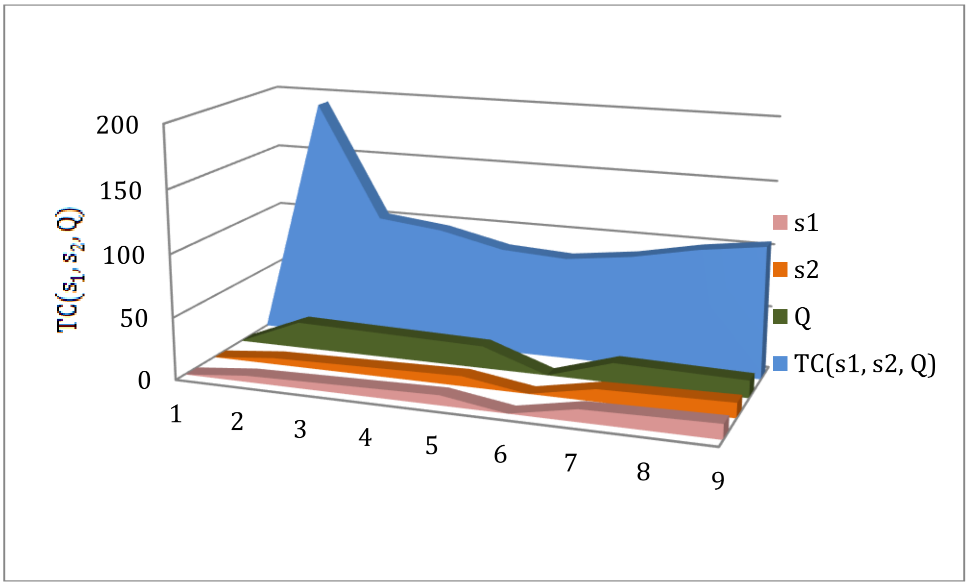
The results we obtained in the steady state case may be illustrated through the following numerical example,

$S_1 = 24$, $S_2 = 24$, M = 120, $\square_1 = 4$, $\square_2 = 2$, $\square_d = 3$, $\square = 3$ $H_1 = 1.5$, $H_2 = 1.6$, $H_d = 1.7$, $O_1 = 2.5$, $O_2 = 2.6$, $O_d = 2.7$, $T_r = 3.4$

The cost for different reorder level are given by

s_1	5	6	7	8	9*	10	11	12
s_2	5	6	7	8	9*	10	11	12
Q	21	20	19	18	17*	16	15	14

$TC(s_1, s_2, Q)$	192.768	102.2113	96.412	84.921	82.0109	88.4173	98.6247	105.66



For the inventory capacity S_1 and S_2, the optimal reorder level s_1, and s_2 and optimal cost TC(s_1, s_2, Q)are indicated by the symbol *. The Convexity of the cost function is given in the graph with common reorder point s (both s_1, and s_2).

Conclusions

This paper presents a substitutable Perishable inventory system in supply chain with three independent demands. The model is analyzed within the framework of Markov processes. Joint probability distributions of inventory levels at DC and Retailer for both products are computed in the steady state. Various system performance measures are derived and the long run expected cost is calculated.

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