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K-Regular Domination In Various Fuzzy Graph

R.Jaya[1] , M. Arockia Ranjithkumar[2] & S.Krishna Kumar

[1]*assistant Professor ,PSNA College Of Engineering And Technology, Dindigul*

[2]*assistant Professor,M. Kumarasamy College Of Engineering, Karur*

[2]*ASSISTANT Professor, Sethu Institute Of Technology, Kariapatti*

EMAIL ID: jayarajaram@psnacet.edu.in[1] & arockiaranjithkumar@gmail.com[2]

Abstract

In this paper we define regular domination set and regular dominating number in fuzzy graph and investigate some properties and bounds of regular domination number in various fuzzy graphs.

Keywords:Fuzzy graph, regular dominating set and regular dominating number.

1. INTRODUCTION:

Rosenfeld introduced the fuzzy graph and define several graph theoretic concepts as path, cycle and connectedness. Mordeson introduced the fuzzy concept of line and developed its basic properties. The domination in fuzzy graph was introduced by Somasundaram. He also investigates the concept of independent domination, total domination and connected domination in fuzzy graphs.

In this paper we define regular domination set and regular dominating number in fuzzy graph and investigate some properties and bounds of regular domination number in various fuzzy graphs.

2. PRELIMINARIES

This section deals the some basic definitions of fuzzy graphs. It is useful to construct the next section.
A fuzzy graph $G(\sigma,\mu)$ is a pair of functions $\sigma: V \to [0,1]$ and $\mu: V\times V \to [0,1]$ where for all $u,v \in V$, we have $\mu(u,v) \le \sigma(u)\Lambda\sigma(v)$.The fuzzy graph $H(\tau,\rho)$ is called a fuzzy subgraph of $G(\sigma,\mu)$ if $\tau(u) \le \sigma(u)$ for all $u \in V$ and $\rho(u,v) \le \mu(u,v)$ for all $u,v \in V$.
For any subset *S* of *V* and let $S \subseteq V$ Fuzzy cardinality of *S* is defined to be $\sum_{v\in S}\sigma(v)$ .The order p and size q of a fuzzy graph $G(\sigma,\mu)$ are defined to be $\sum_{x\in V}\sigma(x)$ and $\sum_{uv\in E}\mu(uv)$ respectively.
The degree of a vertex *v* to is defined by $d(v) = \sum_{u\ne v}\mu(uv)$. The minimum degree of *G* is $\delta(G) = \min\{d(u)/u \in V\}$ and the maximum degree of *G* is $\Delta(G) = \max\{d(u)/u \in V\}$.

An edge of a fuzzy graph is called an effective edge if $\mu(u,v) = \sigma(u)\Lambda\sigma(v)$. $N(u) = \{v \in V / \mu(u,v) = \sigma(u) \wedge \sigma(v)\}$ is called the neighborhood of *u* and $N[u] = N(u) \cup \{u\}$ is called the closed neighborhood of *u*. $d_N(u) = \sum_{v\in N(u)}\sigma(v)$ is called the neighborhood degree of *u*.

The minimum neighborhood degree of G is $\delta_N(G) = \min\{d_N(v)/v \in V\}$ and the maximum neighborhood degree of *G* is $\Delta_N(G) = \max\{d_N(u)/u \in V\}$.

Let G = (V, E) be an IFG. A set $D \subseteq V$ is said to be a dominating set of G if every $v \in V - D$ there exist $u \in D$ such that u dominates v.

Afuzzy dominating set D of a Fuzzy Graphs, G is called minimal dominating set of G if every node $u \in D$, $D - \{u\}$ is not a dominating set in G. Afuzzy domination number $\gamma_{if}(G)$ of a Fuzzy Graphs, G is the minimum vertex cardinality over all minimal dominating sets in G.

3. REGULAR DOMINATING SET

In this section the idea of regular domination in fuzzy graphs and also discusses some properties and bounds of a regular domination number in fuzzy graphs.

Definition 3.1 A set $S \subseteq V$ is said to be a regular dominating set in fuzzy graphs $G(V, E)$ if

i) Every vertex $u \in V - S$ is adjacent to some vertex in S.

ii) Every vertex in $S \subseteq V$ has the same degree.

Minimum cardinality among all the regular dominating sets is called the regular domination number $\gamma_R(G)$ of $G(V, E)$.

Theorem 3.1: In a regular fuzzy graph $G(V, E)$ then every dominating set is a regular dominating set of $G(V, E)$.

Proof: Let $G(V, E)$ be a regular fuzzy graph. Therefore degree of every vertex in $G(V, E)$ are unique. This implies every dominating set is a regular dominating set of $G(V, E)$.

Definition 3.2 . Let $G_1(\sigma_1, \mu_1)$ and $G_2(\sigma_2, \mu_2)$ be two fuzzy graphs on V_1 and V_2 respectively with $V_1 \cap V_2 = \phi$. The union of G_1 and G_2 is the fuzzy graph G on $V_1 \cup V_2$ defined by $G = (G_1 \cup G_2) = (\sigma_1 \cup \sigma_2, \mu_1 \cup \mu_2)$ where

$$(\sigma_1 \cup \sigma_2)(\mathrm{u}) = \begin{Bmatrix} \sigma_1(u) \text{if u} \in \mathrm{V}_1 \\ \sigma_2(u) \, if \; u \in V_2 \end{Bmatrix} \text{and}$$

$$(\mu_1 \cup \mu_2)(\mathrm{uv}) = \begin{Bmatrix} \mu_1(uv) \text{if u, v} \in \mathrm{V}_1 \\ \mu_2(uv) \, if \text{ u, v} \in \mathrm{V}_2 \\ 0 \quad otherwise \end{Bmatrix}$$

Theorem 3.2: Let $G_1(V_1, E_1)$ and $G_2(V_2, E_2)$ are two fuzzy graphs. Let $D_1 \; and \; D_2$ be the minimal K-regular dominating sets of $G_1(V_1, E_1)$ and $G_2(V_2, E_2)$ respectively. Then the regular dominating number of $G_1 \cup G_2$ is $\gamma_R(G_1 \cup G_2) = |D_1| + |D_2|$.

Proof: Let $G_1(V_1, E_1)$ and $G_2(V_2, E_2)$ are two fuzzy graphs. Assume $D_1 \; and \; D_2$ be the minimal regular dominating sets of $G_1(V_1, E_1)$ and $G_2(V_2, E_2)$ respectively. If every vertex $u \in G_1 \cup G_2$ this implies $u \in G_1$ or $u \in G_2$ therefore there is a vertex $v \in D_1$ or $v \in D_2$ such that 'v' regularly dominates $u \in G_1 \cup G_2$. Since $D_1 \; and \; D_2$ be the regular dominating sets of $G_1(V_1, E_1)$ and $G_2(V_2, E_2)$ respectively. The regular dominating number of $G_1 \cup G_2$ is $\gamma_R(G_1 \cup G_2) = |D_1| + |D_2|$. Hence proved.

Example 3.1

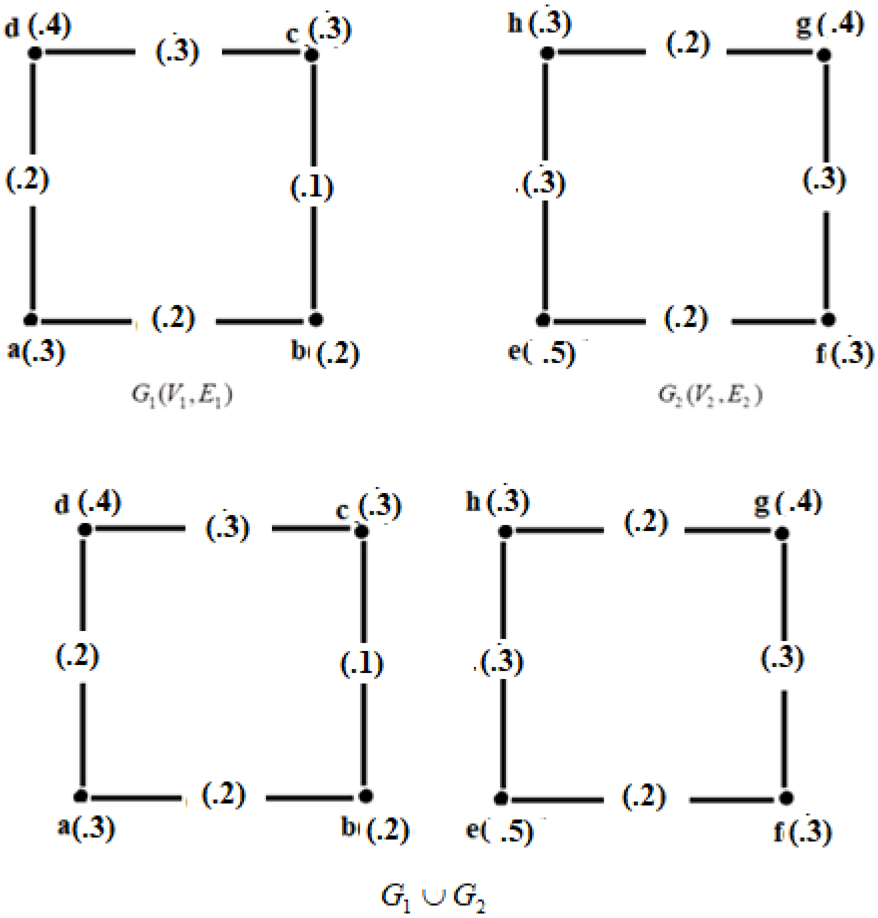


Figure 3.1

In the figure 3.1, the degree of the vertices in $G_1(V_1, E_1)$ and $G_2(V_2, E_2)$ are $d(a) = 0.2$, $d(b) = 0.3$ $d(c) = 0.4$, $d(d) = 0.3$, and $d(e) = 0.3$, $d(f) = 0.4$, $d(g) = 0.3$, $d(h) = 0.5$. The regular dominating set of $G_1(V_1, E_1)$ and $G_2(V_2, E_2)$ are $D_1 = \{b, d\}$ and $D_2 = \{e, g\}$.. The regular dominating set of $(G_1 \cup G_2)$ is $D = \{b, d, e, g\}$ and the minimal dominating number of the graph $(G_1 \cup G_2)$ is $\gamma_{RF}(G_1 \cup G_2) = 1.5$.

Definition 3.3. Let $G_1(\sigma_1, \mu_1)$ and $G_2(\sigma_2, \mu_2)$ is two fuzzy graphs on V_1 and V_2 respectively with $V_1 \cap V_2 = \phi$. The join of G_1 and G_2, denoted by $G_1 + G_2$, is the fuzzy graph on $V_1 \cap V_2$ defined as follows.

$$G_1 + G_2 = (\sigma_1 + \sigma_2, \mu_1 + \mu_2) \text{ Where}$$

$$(\sigma_1 + \sigma_2)(u) = \left\{ \begin{matrix} \sigma_1(u) if\ u \in V_1 \\ \sigma_2(u) if\ u \in V_2 \end{matrix} \right\} \text{and}$$

$$(\mu_1 + \mu_2)(uv) = \left\{ \begin{matrix} \mu_1(uv) if\ u, v \in V_1 \\ \mu_2(uv) if\ u, v \in V_2 \\ \sigma_1(u) \Lambda \sigma_2(v) \text{ if u} \in \text{V}_1\ and\ v \in V_2 \end{matrix} \right\}$$

Theorem 3.3: The sets D_1, D_2 be a regular dominating set of the fuzzy graphs $G_1(V_1, E_1)$ and $G_2(V_2, E_2)$

respectively. If order of $G_1(V_1,E_1)$ and $G_2(V_2,E_2)$ are unique, then $\gamma_R(G_1+G_2)=\min\{|D_1|,|D_2|\}$.

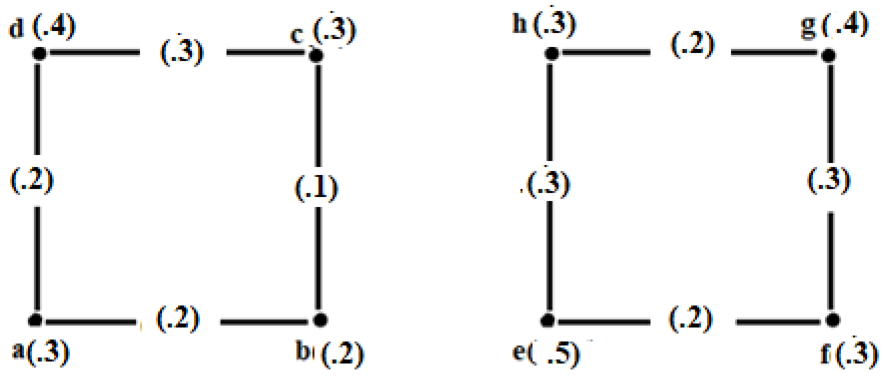
Proof: Let $G_1(V_1,E_1)$ and $G_2(V_2,E_2)$ be fuzzy graphs and The sets D_1,D_2 be regular dominating sets of the fuzzy graphs $G_1(V_1,E_1)$ and $G_2(V_2,E_2)$ respectively. . If order of $G_1(V_1,E_1)$ and $G_2(V_2,E_2)$. This implies we get $O(G_1)=O(G_2)$. In G_1+G_2 every vertex in $G_1(V_1,E_1)$ is adjacent to every vertices in $G_2(V_2,E_2)$ and vise-versa. Thisimplies the sets D_1,D_2 are dominating sets of G_1+G_2 and degree of the vertices in G_1+G_2 are

$$d(v)=\begin{cases}(d_{G_1}(v)+O(G_2)), if\ \ v\in G_1\\(d_{G_2}(v)+O(G_1)), if\ \ v\in G_2\end{cases}$$

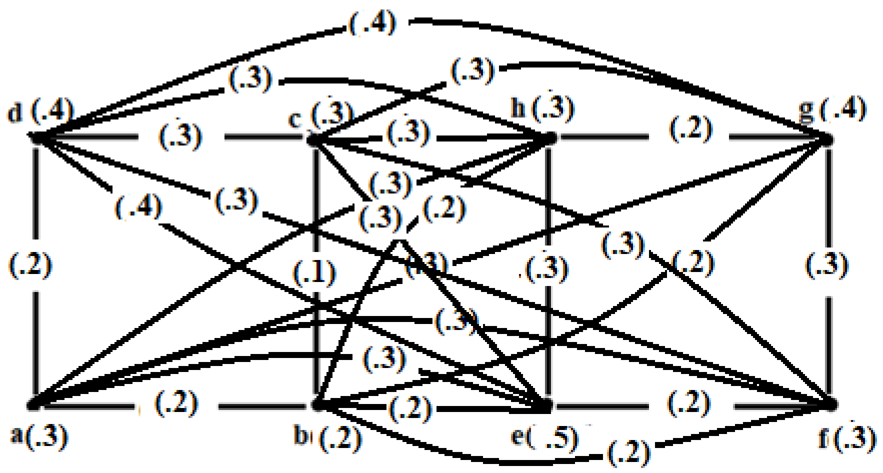
Thisimplies $d(u)=d(v)$, $\forall u,v\in D_1\ or u,v\in D_2$ in G_1+G_2 . Hence $D_1\ or D_2$ be a regular dominating sets of G_1+G_2 . Therefore the minimal dominating number of G_1+G_2 is

$$\gamma_R(G_1+G_2)=\min\{|D_1|,|D_2|\}$$

Example 3.2:



$G_1(V_1,E_1)$ $G_2(V_2,E_2)$



G_1+G_2

Figure 3.3

In the figure 3.2, the degree of the vertices in $G_1(V_1, E_1)$ and $G_2(V_2, E_2)$ are $d(a) = 0.2$, $d(b) = 0.3$ $d(c) = 0.4$, $d(d) = 0.3$, and $d(e) = 0.3$, $d(f) = 0.4$, $d(g) = 0.3$, $d(h) = 0.5$. The regular dominating set of $G_1(V_1, E_1)$ and $G_2(V_2, E_2)$ are $D_1 = \{b, d\}$ and $D_2 = \{e, g\}$ .. The degree of the vertices in $(G_1 + G_2)$ are $d(a) = 1.7$, $d(b) = 1.8$, $d(c) = 1.9$, $d(d) = 1.8$, $d(e) = 1.5$, $d(f) = 1.6$, $d(g) = 1.5$, $d(h) = 1.7$. The graph $(G_1 + G_2)$ does not contain a regular dominating set since $O(G_1) \neq O(G_2)$

Conclusion:

In this paper we define regular domination set and regular domination number in fuzzy graph and investigate some properties and bounds of regular domination number in various fuzzy graphs.

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