On Nano ψ- Closed Sets

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Abstract

we introduce a new class of sets namely nano ψ-closed sets in nano topological spaces. This class lies between the class of nano closed sets and the class of nano g-closed sets. This class also lies between the class of nano closed sets and the class of nano ω -closed sets.

Key words and phrases: nano ψ-closed, nano gs-closed, nano ω-closed.

1. INTRODUCTION

The concept of nano topology was introduced by Lellis Thivagar [4] which was defined in terms of approximations and boundary region of a subset of an universe using an equivalence relation on it. He also established the week forms of nano open sets namely nano α-open sets, nano-semi open sets and nano pre-open sets in a nano topological space.

In this paper, we introduce a new class of sets namely nano ψ-closed sets in nano topological spaces. This class lies between the class of nano closed sets and the class of nano g-closed sets. This class also lies between the class of nano closed sets and the class of nano ω -closed sets.

2. PRELIMINARIES

Throughout this paper $(U, \tau_R(X))$ (or X) represent nano topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a space $(U, \tau_R(X))$, $Ncl(A)$ and $Nint(A)$ denote the nano closure of A and the nano interior of A respectively. We recall the following definitions which are useful in the sequel.

Definition 2.1. [8] Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let $X \subseteq U$.

(1) The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$. That is, $L_R(X) = \bigcup_{x \in U} \{R(x): R(x) \subseteq X\}$, where R(x) denotes the equivalence class determined by x.
(2) The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$. That is, $U_R(X) = \bigcup_{x \in U} \{R(x): R(x) \cap X \neq \phi\}$.
(3) The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not - X with respect to R and it is denoted by $B_R(X)$. That is, $B_R(X) = U_R(X) - L_R(X)$.

Property 2.2. *[4] If (U, R) is an approximation space and $X, Y \subseteq U$; then*

(1) $L_R(X) \subseteq X \subseteq U_R(X)$;

(2) $L_R(\phi) = U_R(\phi) = \phi$ and $L_R(U) = U_R(U) = U$;

(3) $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$;

(4) $U_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y)$;

(5) $L_R(X \cup Y) \supseteq L_R(X) \cup L_R(Y)$;

(6) $L_R(X \cap Y) \subseteq L_R(X) \cap L_R(Y)$;

(7) $L_R(X) \subseteq L_R(Y)$ and $U_R(X) \subseteq U_R(Y)$ whenever $X \subseteq Y$;

(8) $U_R(X^c) = [L_R(X)]^c$ and $L_R(X^c) = [U_R(X)]^c$;

(9) $U_R U_R(X) = L_R U_R(X) = U_R(X)$;

(10) $L_R L_R(X) = U_R L_R(X) = L_R(X)$.

Definition 2.3. [4] Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. Then by the Property 2.2, $\tau_R(X)$ satisfies the following axioms:

(1) U and $\phi \in \tau_R(X)$,

(2) The union of the elements of any sub collection of $\tau_R(X)$ is in $\tau_R(X)$,

(3) The intersection of the elements of any finite subcollection of $\tau_R(X)$ is in $\tau_R(X)$.

That is, $\tau_R(X)$ is a topology on U called the nano topology on U with respect to X. We call $(U, \tau_R(X))$ as the nano topological space. The elements of $\tau_R(X)$ are called as nano open sets and $[\tau_R(X)]^c$ is called as the dual nano topology of $[\tau_R(X)]$.

Remark 2.4. [4] If $[\tau_R(X)]$ is the nano topology on U with respect to X, then the set $B = \{U, \phi, L_R(X), B_R(X)\}$ is the basis for $\tau_R(X)$.

Definition 2.5 [4] If $(U, \tau_R(X))$ is a nano topological space with respect to X and if H $\subseteq$ G, then the nano interior of H is defined as the union of all nano open subsets of H and it is denoted by Nint(H).

That is, Nint(H) is the largest nano open subset of H.

The nano closure of H is defined as the intersection of all nano closed sets containing A and it is denoted by Ncl(H).

That is, Ncl(H) is the smallest nano closed set containing H.

Definition 2.6

A subset H of a space $(U, \tau_R(X))$ is called:

(i) nano semi-open set [4] if H $\subseteq$ Ncl(Nint(H));

(ii) nano preopen set [4] if H $\subseteq$ Nint(Ncl(H));

(iii) nano α -open set [4] if H $\subseteq$ Nint(Ncl(Nint(H)));

(iv) nano β-open set [6] (= nano semi-preopen) if H $\subseteq$ Ncl(Nint(Ncl(H)));

(v) nano regular open set [4] if H = Nint(Ncl(H)).

The complements of the above mentioned open sets are called their respective closed sets.

The nano preclosure [4] (resp. nano semi-closure [4], nano α -closure [4], nano semi-pre-closure [6]) of a subset H of X, denoted by Npcl(H) (resp. Nscl(H), Nα cl(H), Nspcl(H)) is defined to be the intersection of all nano preclosed (resp. nano semi-closed, nanoα -closed, nano semi-preclosed) sets of $(U, \tau_R(X))$ containing H. It is known that Npcl(H) (resp. Nscl(H), Nα cl(H), Nspcl(H)) is a nano preclosed (resp. nano semi-closed, nanoα -closed, nano semi-preclosed) set. For any subset H of an arbitrarily chosen nano topological space, the nano semi-interior [4] (resp. nanoα -interior [4], nano preinterior [4]) of H, denoted by Nsint(H) (resp. Nα int(H), Npint(H)), is defined to be the union of all nano semi-open (resp. nanoα -open, nano preopen) sets of $(U, \tau_R(X))$ contained in H.

Definition 2.7

A subset H of a space $(U, \tau_R(X))$ is called:

(i) a nano generalized closed (briefly nano g-closed) set [2] if Ncl(H) $\subseteq$ G whenever H $\subseteq$ G and U is nano open in $(U, \tau_R(X))$. The complement of nano g-closed set is called nano g-open set;

(ii) a nano semi-generalized closed (briefly nano sg-closed) set [1] if Nscl(H) $\subseteq$ G whenever H $\subseteq$ G and G is semi-open in $(U, \tau_R(X))$. The complement of nano sg-closed set is called nano sg-open set;

(iii) a nano generalized semi-closed (briefly nano gs-closed) set [1] if Nscl(A) $\subseteq$ G whenever H $\subseteq$ G and G is nano open in $(U, \tau_R(X))$. The complement of nano gs-closed set is called nano gs-open set;

(iv) a nanoα -generalized closed (briefly nanoα g-closed) set [7] if Nα cl(H) $\subseteq$ G whenever H $\subseteq$ G and G is nano open in $(U, \tau_R(X))$. The complement of nanoα g-closed set is called nanoα g-open set;

(v) a nano generalized semi-preclosed (briefly nano gsp-closed) set [10] if Nspcl(H) $\subseteq$ G whenever H $\subseteq$ G and G is nano open in $(U, \tau_R(X))$. The complement of nano gsp-closed set is called nano gsp-open set;

(vi) a nano ĝ-closed set [5] (=nano ω -closed) if Ncl(H) $\subseteq$ G whenever H $\subseteq$ G and G is nano semi-open in $(U, \tau_R(X))$. The complement of nano ĝ-closed set is called nano ĝ-open set;

(vii) a nano g*s-closed set [9] if Nscl(H) $\subseteq$ G whenever H $\subseteq$ G and G is nano gs-open in $(U, \tau_R(X))$. The complement of nano g*s-closed set is called nano g*s-open set;

Remark 2.8

The collection of all nano ω -closed (resp. nano g-closed, nano gs-closed, nano gsp-closed, nano α g-closed, nano sg-closed, nano g*s-closed, nano α -closed, nano semi-closed, nano regular closed) sets is denoted by NωC(U) (resp. NGC(U), NGSC(U), NGSPC(U), NαgC(U), NSGC(U), NG*SC(U), NαC(U), NSC(U), NRC(U)).

The collection of all nano ω-open (resp. nano g-open, nano gs-open, nano gsp-open, nano α g-open, nano sg-open, nano g*s-open, nano α-open, nano semi-open, nano regular open) sets is denoted by NωO(U) (resp. NGO(U), NGSO(U), NGSPO(U), NαgO(U), NSGO(U), NG*SO(U), NαO(U), NSO(U), NRO(U)).

We denote the power set of U by P(U).

Definition 2.9 [3]

A subset H of (U, τ_R(X)) is said to be nano locally closed if H = G ∩ F, where G is nano open and F is nano closed in $(U, \tau_R(X))$.

Remark 2.10

(i) Every nano open set is nano g*s-open [9].

(ii) Every nano semi-open set is nano g*s-open [9].

(iii) Every nano g*s-open set is nano sg-open [9].

(iv) Every nano semi-closed set is nano gs-closed [1].

(v) Every nano closed set is nano gs-closed [1].

Corollary 2.11 [1]

Let H be both nano open and nano sg-closed set and suppose that F is nano closed set. Then H ∩ F is nano gs-closed set.

3. NANO ψ-CLOSED SETS

We introduce the following definition.

Definition 3.1

(i) A subset H of (U, τ_R(X)) is called a nano ψ-closed set if Ncl(H) ⊆ G whenever H ⊆ G and G is nano gs-open in $(U, \tau_R(X))$. The complement of nano ψ-closed set is nano ψ-open

The collection of all nano ψ-closed (resp. nano ψ-open) sets is denoted by NψC(U) (resp. NψO(U)).

Proposition 3.2

In a nano topological space $(U, \tau_R(X))$, every nano closed set is nano ψ-closed.

Proof

If H is any nano closed set in $(U, \tau_R(X))$ and G is any nano gs-open set containing H, then G ⊇ H = Ncl(H). Hence H is nano ψ-closed.

The converse of Proposition 3.2 need not be true as seen from the following example.

Example 3.3

Let U = {1, 2, 3} with U/R = {{1}, {2, 3}} and X = {1, 2}. Then the nano topology τ_R(X) = {ϕ, U, {1, 2}}. Then NψC(U) = {ϕ, {3}, {1, 3}, {2, 3}, U}. Here, H = {1, 3} is nano ψ-closed set but not nano closed.

Proposition 3.4

In a nano topological space $(U, \tau_R(X))$, every nano ψ-closed set is nano g*s-closed.

Proof

If H is a nano ψ-closed subset of $(U, \tau_R(X))$ and G is any nano gs-open set containing H, then G ⊇ Ncl(H) ⊇ Nscl(H). Hence H is nano g*s-closed in $(U, \tau_R(X))$.

The converse of Proposition 3.4 need not be true as seen from the following example.

Example 3.5

Let U = {1, 2, 3} with U/R = {{1}, {2, 3}} and X = {2}. Then the nano topology $\tau_R(X)$ = {ϕ, U, {2}}. Then NG*SC(U) = {ϕ, {1}, {3}, {1, 3}, U}. Here, H = {3} is nano g*s-closed but not nano ψ-closed set in $(U, \tau_R(X))$.

Proposition 3.6

In a nano topological space $(U, \tau_R(X))$, every nano ψ-closed set is nano ω-closed.

Proof

Suppose that H ⊆ G and G is nano semi-open in $(U, \tau_R(X))$. Since every nano semi-open set is nano gs-open and H is nano ψ-closed, therefore Ncl(H) ⊆ G. Hence H is nano ω-closed in $(U, \tau_R(X))$.

The converse of Proposition 3.6 need not be true as seen from the following example.

Example 3.7

Let U = {1, 2, 3} with U/R = {{1}, {2, 3}} and X = {1, 3}. Then the nano topology $\tau_R(X)$ = {ϕ, U, {1}, {2, 3}}. Then NψC(U) = {ϕ, {1}, {2, 3}, U} and NωC(U) = P(U). Here, H = {1, 3} is nano ω-closed but not nano ψ-closed set in $(U, \tau_R(X))$.

Proposition 3.8

In a nano topological space $(U, \tau_R(X))$, every nano g*s-closed set is nano sg-closed.

Proof

Suppose that H ⊆ G and G is nano semi-open in $(U, \tau_R(X))$. Since every nano semi-open set is nano gs-open and H is nano g*s-closed, therefore Nscl(H) ⊆ G. Hence H is nano sg-closed in $(U, \tau_R(X))$.

The converse of Proposition 3.8 need not be true as seen from the following example.

Example 3.9

Let U = {1, 2, 3} with U/R = {{1}, {2, 3}} and X = {1, 3}. Then the nano topology $\tau_R(X)$ = {ϕ, U, {1}, {2, 3}}. Then NG*SC(U) = {ϕ, {1}, {2, 3}, U} and NSGC(U) = P(U). Here, H = {1, 2} is nano sg-closed but not nano g*s-closed set in $(U, \tau_R(X))$.

Proposition 3.10

In nano topological space $(U, \tau_R(X))$, every nano ψ-closed set is nano g-closed.

Proof

If H is a nano ψ-closed subset of $(U, \tau_R(X))$ and G is any nano open set containing H, since every nano open set is nano gs-open, we have G ⊇ Ncl(H). Hence H is nano g-closed in $(U, \tau_R(X))$.

The converse of Proposition 3.10 need not be true as seen from the following example.

Example 3.11

Let U = {1, 2, 3} with U/R = {{1}, {2, 3}} and X = {1, 3}. Then the nano topology $\tau_R(X)$ = {ϕ, U, {1}, {2, 3}}. Then NψC(U) = {ϕ, {1}, {2, 3}, U} and NGC(U) = P(U). Here, H = {1, 2} is nano g-closed but not nano ψ-closed set in $(U, \tau_R(X))$.

Proposition 3.12

In a nano topological space $(U, \tau_R(X))$, every nano ψ-closed set is nano α g-closed.

Proof

If H is a nano ψ-closed subset of $(U, \tau_R(X))$ and G is any nano open set containing H, since every nano open set is nano gs-open, we have G ⊇ Ncl(H) ⊇ Nα cl(H). Hence H is nano α g-closed in $(U, \tau_R(X))$.

The converse of Proposition 3.12 need not be true as seen from the following example.

Example 3.13

Let U = {1, 2, 3} with U/R = {{1, 2}, {3}} and X = {2, 3}. Then the nano topology $\tau_R(X)$ = {ϕ, U, {3}, {1, 2}}. Then NψC(U) = {ϕ, {3}, {1, 2}, U} and NαgC(U) = P(U). Here, H = {1, 3} is nano α g-closed but not nano ψ-closed set in $(U, \tau_R(X))$.

Proposition 3.14

In a nano topological space $(U, \tau_R(X))$, every nano ψ-closed set is nano gs-closed.

Proof

If H is a nano ψ-closed subset of $(U, \tau_R(X))$ and G is any nano open set containing H, since every nano open set is nano gs-open, we have G ⊇ Ncl(H) ⊇ Nscl(H). Hence H is nano gs-closed in $(U, \tau_R(X))$.

The converse of Proposition 3.14 need not be true as seen from the following example.

Example 3.15

Let U = {1, 2, 3} with U/R = {{1}, {2, 3}} and X = {1}. Then the nano topology $\tau_R(X)$ = {ϕ, U, {1}}. Then NψC(U) = {ϕ, {2, 3}, U} and NGSC(U) = {ϕ, {2}, {3}, {1, 2}, {1, 3}, {2, 3}, U}. Here, H = {3} is nano gs-closed but not nano ψ-closed set in $(U, \tau_R(X))$.

Proposition 3.16

In a nano topological space $(U, \tau_R(X))$, every nano ψ-closed set is nano gsp-closed.

Proof

If H is a nano ψ-closed subset of $(U, \tau_R(X))$ and G is any nano open set containing H, every nano open set is nano gs-open, we have G ⊇ Ncl(H) ⊇ Nspcl(H). Hence H is nano gsp-closed in $(U, \tau_R(X))$.

The converse of Proposition 3.16 need not be true as seen from the following example.

Example 3.17

In Example 3.15, NGSPC(U) = {ϕ, {2}, {3}, {1, 2}, {1, 3}, {2, 3}, U}. Here, H = {3} is nano gsp-closed but not nano ψ-closed set in $(U, \tau_R(X))$.

Remark 3.18

The following example shows that nano ψ-closed sets are independent of nano α-closed sets and nano semi-closed sets.

Example 3.19

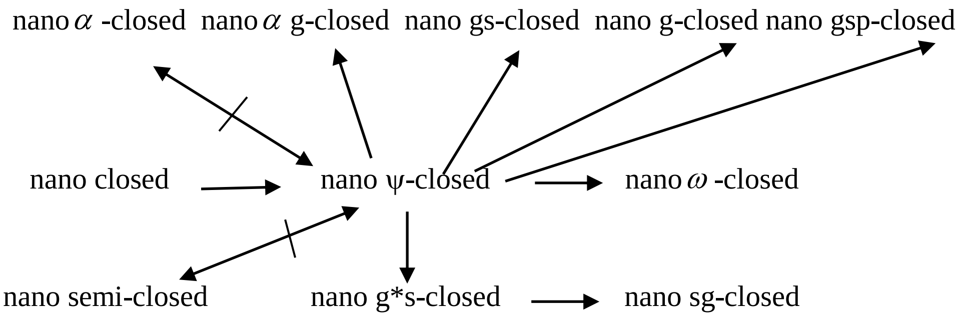
Let U = {1, 2, 3} with U/R = {{1}, {2, 3}} and X = {1, 2}. Then the nano topology $\tau_R(X)$ = {ϕ, U, {1, 2}}. Then NψC(U) = {ϕ, {3}, {1, 3}, {2, 3}, U} and NαC(U) = NSC(U) = {ϕ, {3}, U}. Here, H = {1, 3} is nano ψ-closed but it is neither nano α-closed nor nano semi-closed in $(U, \tau_R(X))$.

Example 3.20

Let U = {1, 2, 3} with U/R = {{1}, {2, 3}} and X = {1}. Then the nano topology $\tau_R(X)$ = {ϕ, U, {1}}. Then NψC(U) = {ϕ, {2, 3}, U} and NαC(U) = NSC(U) = {ϕ, {2}, {3}, {2, 3}, U}. Here, H = {2} is nano α-closed as well as nano semi-closed in $(U, \tau_R(X))$ but it is not nano ψ-closed in $(U, \tau_R(X))$.

Remark 3.21

From the above discussions and known results in [86, 99, 113], we obtain the following diagram, where H → K (resp. H ⟷ K) represents A implies B but not conversely (resp. A and B are independent of each other).



None of the above implications are reversible.

4. PROPERTIES OF NANO ψ-CLOSED SETS

In this section, we discuss some basic properties of nano ψ-closed sets.

Definition 4.1

The intersection of all nano gs-open subsets of $(U, \tau_R(X))$ containing H is called the nano gs-kernel of A and denoted by Ngs-ker(H).

Lemma 4.2

A subset H of $(U, \tau_R(X))$ is nano ψ-closed if and only if Ncl(A) ⊆ Ngs-ker(H).

Proof

Suppose that H is nano ψ-closed. Then Ncl(H) ⊆ G whenever H ⊆ G and G is nano gs-open. Let x ∈ Ncl(H). If x ∉ Ngs-ker(H), then there is a nano gs-open set G containing H such that x ∉ G. Since G is a nano gs-open set containing H, we have x ∉ Ncl(H) and this is a contradiction.

Conversely, let Ncl(H) ⊆ Ngs-ker(H). If G is any nano gs-open set containing H, then Ncl(H) ⊆ Ngs-ker(H) ⊆ G. Therefore, H is nano ψ-closed.

Corollary 4.3

If H is a nano ψ-closed set and F is a nano closed set, then H ∩ F is a nano ψ-closed set.

Proposition 4.4

If H and K are nano ψ-closed sets in $(U, \tau_R(X))$, then H ∪ K is nano ψ-closed in $(U, \tau_R(X))$.

Proof

If H ∪ K ⊆ G and G is nano gs-open, then H ⊆ G and K ⊆ G. Since H and K are nano ψ-closed, G ⊇ Ncl(H) and G ⊇ Ncl(K) and hence G ⊇ Ncl(H) ∪ Ncl(K) = Ncl(H ∪ K). Thus H ∪ K is nano ψ-closed set in $(U, \tau_R(X))$.

Proposition 4.5

If a set H is nano ψ-closed in $(U, \tau_R(X))$, then Ncl(H) – H contains no nonempty nano closed set in $(U, \tau_R(X))$.

Proof

Suppose that H is nano ψ-closed. Let F be a nano closed subset of Ncl(H) – H. Then H ⊆ F^c. But H is nano ψ-closed, therefore Ncl(H) ⊆ F^c. Consequently, F ⊆ $(Ncl(H))^c$. We already have F ⊆ Ncl(H). Thus F ⊆ Ncl(H) ∩ $(Ncl(H))^c$ and F is empty.

The converse of Proposition 4.5 need not be true as seen from the following example.

Example 4.6

Let U = {1, 2, 3} with U/R = {{1}, {2, 3}} and X = {1}. Then the nano topology $\tau_R(X)$ = {ϕ, U, {1}}. Then NψC(U) = {ϕ, {2, 3}, U}. If H = {2}, then Ncl(H) – H = {3} does not contain any nonempty nano closed set. But H is not nano ψ-closed in $(U, \tau_R(X))$.

Theorem 4.7

A set H is nano ψ-closed if and only if Ncl(A) – A contains no nonempty nano gs-closed set.

Proof

Necessity. Suppose that H is nano ψ-closed. Let K be a nano gs-closed subset of Ncl(H) – H. Then H ⊆ K^c. Since H is nano ψ-closed, we have Ncl(H) ⊆ K^c. Consequently, K ⊆ $(Ncl(H))^c$. Hence, K ⊆ Ncl(H) ∩ $(Ncl(H))^c$ = ϕ. Therefore K is empty.

Sufficiency. Suppose that Ncl(H) – H contains no nonempty nano gs-closed set. Let H ⊆ G and G be both nano closed and nano sg-open. If Ncl(H) ⊄ G, then Ncl(H) ∩ G^c ≠ ϕ. Since Ncl(H) is a nano closed set and G^c is both nano open and nano sg-closed set, Ncl(H) ∩ G^c is a nonempty nano gs-closed subset of Ncl(H) – H . This is a contradiction. Therefore, Ncl(H) ⊆ G and hence H is nano ψ-closed.

Proposition 4.8

If H is nano ψ-closed in $(U, \tau_R(X))$ and H ⊆ K ⊆ Ncl(H), then K is nano ψ-closed in $(U, \tau_R(X))$.

Proof

Let K ⊆ G where G is nano gs-open in $(U, \tau_R(X))$. Since H ⊆ K, H ⊆ G. Since H is nano ψ-closed in $(U, \tau_R(X))$, Ncl(H) ⊆ G whenever H ⊆ G and G is nano gs-open in $(U, \tau_R(X))$. Since K ⊆ Ncl(H), Ncl(K) ⊆ Ncl(H) ⊆ G. Hence K is nano ψ-closed set in $(U, \tau_R(X))$.

Proposition 4.9

If H is a nano gs-open and nano ψ-closed in $(U, \tau_R(X))$, then H is nano closed in $(U, \tau_R(X))$.

Proof

Since H is nano gs-open and nano ψ-closed, Ncl(H) ⊆ H and hence H is nano closed in $(U, \tau_R(X))$.

Recall that a nano topological space $(U, \tau_R(X))$ is called extremally disconnected if Ncl(G) is nano open for each G ∈ τ_R(X).

Theorem 4.10

Let $(U, \tau_R(X))$ be extremally disconnected and H is a nano semi-open subset of U. Then H is nano ψ-closed if and only if it is nano gs-closed.

Proof

It follows from the fact that if $(U, \tau_R(X))$ is extremally disconnected and H is a nano semi-open subset of U, then Nscl(H) = Ncl(H).

Theorem 4.11

Let H be a nano locally closed set of $(U, \tau_R(X))$. Then H is nano closed if and only if A is nano ψ-closed.

Proof

(i) ⇒ (ii). It is fact that every nano closed set is nano ψ-closed.

(ii) ⇒ (i). Now H ∪ (U – Ncl(H)) is nano open in $(U, \tau_R(X))$, since H is nano locally closed. Now H ∪ (U – Ncl(H)) is nano gs-open set of $(U, \tau_R(X))$ such that H ⊆ H ∪ (U – Ncl(H)). Since H is nano ψ-closed, then Ncl(H) ⊆ H ∪ (U – Ncl(H)). Thus, we have Ncl(H) ⊆ H and hence H is a nano closed.

Proposition 4.12

For each x ∈ U, either {x} is nano gs-closed or $\{x\}^c$ is nano ψ-closed in $(U, \tau_R(X))$.

Proof

Suppose that {x} is not nano gs-closed in $(U, \tau_R(X))$. Then $\{x\}^c$ is not nano gs-open and the only nano gs-open set containing $\{x\}^c$ is the space U itself. Therefore Ncl($\{x\}^c$) ⊆ U and so $\{x\}^c$ is nano ψ-closed in $(U, \tau_R(X))$.

Theorem 4.13

Let H be a nano ψ-closed set of a nano topological space $(U, \tau_R(X))$. Then,

(i) Nsint(H) is nano ψ-closed.

(ii) If H is nano regular open, then Npint(H) and Nscl(H) are also nano ψ-closed sets.

(iii) If H is nano regular closed, then Npcl(H) is also nano ψ-closed.

Proof

(i) Since Ncl(Nint(H)) is a nano closed set in $(U, \tau_R(X))$, by Corollary 4.3, Nsint(H) = H ∩ Ncl(Nint(H)) is nano ψ-closed in $(U, \tau_R(X))$.

(ii) Since H is nano regular open in U, H = Nint(Ncl(H)). Then Nscl(H) = H ∪ Nint(Ncl(H)) = H. Thus, Nscl(H) is nano ψ-closed in $(U, \tau_R(X))$. Since Npint(H) = H ∩ Nint(Ncl(H)) = H, Npint(H) is nano ψ-closed.

(iii) Since H is nano regular closed in U, H = Ncl(Nint(H)). Then Npcl(H) = H ∪ Ncl(Nint(H)) = H. Thus, Npcl(H) is nano ψ-closed in $(U, \tau_R(X))$.

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