On Fundamental Algebraic Structures on Direct Product of Complex Anti $\omega -Q -$Fuzzy Subrings

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Abstract

On this paper, we introduce idea of Cartesian product of π-Complex Anti $\omega - Q - fuzzy\ sets$ and discussed with various algebraic aspects. We show that essential Algebraic systems on Direct Product of Complex Anti $\omega - Q -$Fuzzy Subrings and their results.

Keywords: Fuzzy Set, Complex Fuzzy set, Fuzzy Subring, Q-Fuzzy Subring, π-complex anti $\omega - Q -$fuzzy sets and complex anti $\omega - Q -$ fuzzy subrings.

I Introduction

The idea of fuzzy sets turned into added means by Zadeh [10] in 1965. Bhakat S K et.al.[1], defined the belief of Fuzzy subrings and ideals redefined in 1996. In 1990, Fuzzy subgroups and anti Fuzzy subgroups were initiated by Biswas R[2]. Buckley J J[3], commenced the idea of fuzzy complex numbers in 1989. Muhammad et.al[4], proposed the idea of On a few characterization of Q-complex fuzzy sub-rings in 2021. In 2002, commenced new concept of Complex fuzzy sets by Ramot D et.al[5]. Solairaju A and Nagarajan R [8] explored a new structure and construction of Q-fuzzy groups in 2009. Sither Selvam P M et al. [9] described the notion of some properties of anti Q-fuzzy subgroups in 2014. Rasuli R [7] discussed Q-fuzzy subring with respect to t-norm in 2018. Zhang G Q [11], explored a new structure and construction of operation properties and δ-equalities of complex fuzzy sets. In 2003, Complex fuzzy logic brought by way of Ramot D et.al[6].

In this paper, we define the Cartesian product of π-complex anti $\omega - Q -$ fuzzy sets and prove that the results. We also define Cartesian product of complex anti $\omega - Q -$ fuzzy subrings and discuss its properties.

II Preliminaries

Definition 2.1 [10]:

A fuzzy set A of a nonempty set P is a mapping

$$A : P \rightarrow [0, 1].$$

Definition 2.2 [1]:

A fuzzy set A of a ring S is called a FSR of S if

1. $A(m - n) \geq \min\{A(m), A(n)\}, \quad \forall\, m, n \in S$
2. $A(mn) \geq \min\{A(m), A(n)\}, \quad \forall\, m, n \in S$

Definition 2.3 [8]:

Let Q and S be any two sets. Then the mapping $A: S \times Q \to [0,1]$ is called a Q-Fuzzy set in S

Definition 2.4 [7]:

Let Q-Fuzzy set A of ring S is said to be Q-Fuzzy subring if the following conditions are,

1. $A(m-n,q) \geq \min\{A(m,q), A(n,q)\}$, for all $m,n \in S$ and $q \in Q$.
2. $A(mn,q) \geq \min\{A(m,q), A(n,q)\}$, for all $m,n \in S$ and $q \in Q$

Definition 2.5 [5]:

A complex fuzzy set A of universe of discourse P is identify with the membership function $\theta_A(m) = \eta_A(m)e^{i\varphi_A(m)}$ and is defined as

$$\theta_A: P \to \{z \in C: |z| \leq 1\}$$

This membership function receive all membership value from the unit disc on plane, where $i = \sqrt{-1}$, both $\eta_A(m)$ and $\varphi_A(m)$ are real valued such that $\eta_A(m) \in [0,1]$ and $\varphi_A(m) \in [0,2\pi]$.

Definition 2.6 [11]:

Let A and B two complex fuzzy sets of set P. The Cartesian product of complex fuzzy sets A and B is defined as

$$\theta_{A\times B}(m,n) = \eta_{A\times B}(m,n)e^{i\varphi_{A\times B}(m,n)} = \min\{\eta_A(m), \eta_B(n)\}e^{i\min\{\varphi_A(m),\varphi_B(n)\}}$$

Definition 2.7 [2]:

Let A be fuzzy subset of a group H. Then A is said to an anti-fuzzy subgroup if $A(u^{-1}v) \leq \max\{A(u), A(v)\}$, for all $u,v \in H$.

Definition 2.8 [9]:

A function $A: H \times Q \longrightarrow [0,1]$ is a anti-QFSG of a group H if $A(uv^{-1},q) \leq \max\{A(u,q), A(v,q)\}$, for all $u,v \in H$ and $q \in Q$.

III Fundamental Algebraic Structures on Direct Product of Complex anti $\omega - Q$ −Fuzzy Subrings

In this content, We use the concept of complex anti $\omega - Q$ −fuzzy subring to outline direct product of π-complex anti $\omega - Q$ −fuzzy subring . We prove that Cartesian product of two complex anti $\omega - Q$ −fuzzy subring is complex anti $\omega - Q$ −fuzzy subring and illustrate their results.

Definition: 4.1

Let S and Q be any two nonempty sets and $\omega \in [0,1]$ and A be a $Q -$ Fuzzy subset of a set G. The fuzzy set A^ω of G is called the Anti $\omega - Q -$ Fuzzy subset of G is defined by

$$A^\omega(\theta,q) = max\{A(\theta,q), \omega\}, \forall\, \theta \in S \; and \; q \in Q.$$

Definition 4.2:

Let A and B be any two π-complex anti $\omega - Q$ −fuzzy sets of sets S_1 and S_2 respectively. The Cartesian product of π-complex anti $\omega - Q -$ fuzzy sets A^ω and B^ω is defined as $A^\omega{}_\pi \times B^\omega{}_\pi((m,n),q) = \max\{A^\omega{}_\pi(m,q), B^\omega{}_\pi(n,q)\}$, for all $m \in S_1 and\; n \in S_2$ and $q \in Q$

Note:

Let A^ω and B^ω be two π-Q- complex anti $\omega - Q$ −fuzzy subring of S_1 and S_2, respectively. Then $A^\omega \times B^\omega$ is π-anti $\omega - Q -$ fuzzy subring of $S_1 \times S_2$.

Definition 4.3

Let A^ω and B^ω two complex anti $\omega - Q$ −fuzzy subring of sets S_1 and S_2. The Cartesian product of complex anti $\omega - Q -$ fuzzy subrings A^ω and B^ω is defined by a function

$$\theta_{A^\omega \times B^\omega}((m,n),q) = \eta_{A^\omega\times B^\omega}((m,n),q)e^{i\varphi_{A^\omega\times B^\omega}((m,n),q)}$$
$$= \max\{\eta_{A^\omega}(m,q), \eta_{B^\omega}(n,q)\}e^{i\max\{\varphi_{A^\omega}(m,q),\varphi_{B^\omega}(n,q)\}}$$

Theorem 4.4:

Let A^ω and η_{B^ω} be two complex anti $\omega - Q$ −fuzzy subrings of S_1 and S_2 respectively. Then $A^\omega \times B^\omega$ is complex anti $\omega - Q$ −fuzzy subring of $S_1 \times S_2$.

Proof: Let $m, x \in S_1$ and $n, y \in S_2$ be an elements and $q \in Q$. Then $(m,n),(x,y) \in S_1 \times S_2$. Consider,

$$\begin{aligned}
\theta_{A^\omega \times B^\omega}\left((m,n)-(x,y),q\right) &= \theta_{A^\omega \times B^\omega}\left((m-x,n-y),q\right)\\
&= \eta_{A^\omega \times B^\omega}\left((m-x,n-y),q\right)e^{i\varphi_{A^\omega \times B^\omega}((m-x,n-y),q)}\\
&= \max\{\eta_{A^\omega}(m-x,q),\eta_{B^\omega}(n-y,q)\}\, e^{i\max\{\varphi_{A^\omega}(m-x,q),\varphi_{B^\omega}(n-y,q)\}}\\
&= \max\{\eta_{A^\omega}(m-x,q)e^{i\varphi_{A^\omega}(m-x,q)},\eta_{B^\omega}(n-y,q)e^{i\varphi_{A^\omega}(n-y,q)}\}\\
&= \max\{\theta_{A^\omega}(m-x,q),\theta_{B^\omega}(n-y,q)\}\\
&\leq \max\{\max\{\theta_{A^\omega}(m,q),\theta_{A^\omega}(x,q)\},\max\{\theta_{B^\omega}(n,q),\theta_{B^\omega}(y,q)\}\}\\
&= \min\{\max\{\theta_{A^\omega}(m,q),\theta_{B^\omega}(n,q)\},\max\{\theta_{A^\omega}(x,q),\theta_{B^\omega}(y,q)\}\}
\end{aligned}$$

Thus, $\theta_{A^\omega \times B^\omega}\left((m,n)-(x,y),q\right) \leq \max\{\theta_{A^\omega \times B^\omega}\left((m,n),q\right),\theta_{A^\omega \times B^\omega}\left((x,y),q\right)\}$

Further,

$$\begin{aligned}
\theta_{A^\omega \times B^\omega}\left((m,n)(x,y),q\right) &= \theta_{A^\omega \times B^\omega}\left((mx,ny),q\right)\\
&= \eta_{A^\omega \times B^\omega}\left((mx,ny),q\right)e^{i\varphi_{A^\omega \times B^\omega}((mx,ny),q)}\\
&= \max\{\eta_{A^\omega}(mx,q),\eta_{B^\omega}(ny,q)\}\, e^{i\max\{\varphi_{A^\omega}(mx,q),\varphi_{B^\omega}(ny,q)\}}\\
&= \max\{\eta_{A^\omega}(mx,q)e^{i\varphi_{A^\omega}(mx,q)},\eta_{B^\omega}(ny,q)e^{i\varphi_{A^\omega}(ny,q)}\}\\
&= \max\{\theta_{A^\omega}(mx,q),\theta_{B^\omega}(ny,q)\}\\
&\leq \max\{\max\{\theta_{A^\omega}(m,q),\theta_{A^\omega}(x,q)\},\max\{\theta_{B^\omega}(n,q),\theta_{B^\omega}(y,q)\}\}\\
&= \max\{\max\{\theta_{A^\omega}(m,q),\theta_{B^\omega}(n,q)\},\max\{\theta_{A^\omega}(x,q),\theta_{B^\omega}(y,q)\}\}
\end{aligned}$$

Therefore, $\theta_{A^\omega \times B^\omega}\left((m,n)(x,y),q\right) \leq \max\{\theta_{A^\omega \times B^\omega}\left((m,n),q\right),\theta_{A^\omega \times B^\omega}\left((x,y),q\right)\}$

Thus conclude the proof.

Corollary 4.5:

Let ${A^\omega}_1, {A^\omega}_2 \dots {A^\omega}_n$ be complex anti $\omega - Q$ −fuzzy subrings of $S_1, S_2, \dots S_n$ respectively. Then ${A^\omega}_1 \times {A^\omega}_2 \times \dots \times {A^\omega}_n$ is complex anti $\omega - Q$ −fuzzy subring of $S_1 \times S_2 \times \dots \times S_n$.

Remark 4.6:

Let ${A^\omega}_1 and\, {A^\omega}_2$ be two complex anti $\omega - Q$ −fuzzy subrings of S_1 and S_2 respectively and ${A^\omega}_1 and\, {A^\omega}_2$ be complex anti $\omega - Q$ −fuzzy subring of $S_1 \times S_2$. Then it not compulsory both ${A^\omega}_1 and\, {A^\omega}_2$ should be complex anti $\omega - Q$ −fuzzy subring of S_1 and S_2 respectively.

Example 4.7:

Let $\overline{Z}_2 = \{0,1\}$ and $S = \{e,a,b,c\}$ be two rings. Where S is ring and 2×2 matrices over $\overline{Z}_2$ with 2nd row has 0. where $= \begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}, a = \begin{bmatrix}0 & 1\\0 & 0\end{bmatrix}, b = \begin{bmatrix}1 & 0\\0 & 0\end{bmatrix}, c = \begin{bmatrix}1 & 1\\0 & 0\end{bmatrix}$,.

$\overline{Z}_2 \times S = \{(0,e),(0,a),(0,b),(0,c),(1,e),(1,a),(1,b),(1,c)\}$. Then two $\omega - Q$-CFSRs A_1 and A_2 of $\overline{Z}_2$ and S is defined by

$$A_1 = \left\{\left((0,q), 0.3e^{i\frac{\pi}{12}}\right),\left((1,q), 0.2e^{i\frac{\pi}{15}}\right)\right\}, \text{ where } q \in Q$$

$$A_2 = \left\{\left((e,q), 0.4e^{i\frac{\pi}{3}}\right),\left((a,q), 0.55e^{i\frac{\pi}{2}}\right),\left((a^2,q), 0..43e^{i\frac{\pi}{3}}\right),\left((a^3,q), 0.5e^{i\pi}\right)\right\}$$

$$(A_1 \times A_2)(m,q) = \begin{cases}0.3e^{i\frac{\pi}{12}}, & \text{for all } m \in \{(0,e),(0,a),(0,b),(0,c)\}\\ 0.2e^{i\frac{\pi}{15}}, & \text{for all } m \in \{(1,e),(1,a),(1,b),(1,c)\}\end{cases}$$

Here, ${A^\omega}_1 \times {A^\omega}_2$ is complex anti $\omega - Q$ −fuzzy subring of $Z_2 \times S$ and ${A^\omega}_1$ is complex anti $\omega - Q$ −fuzzy subring of Z_2. But ${A^\omega}_2$ is not a complex anti $\omega - Q$ −fuzzy subring of S.

of H_1. But ${A^\omega}_2$ is not a complex anti $\omega - Q$ −fuzzy subring of S_2.

Theorem 4.8:

Let $A^\omega and\, B^\omega$ be two complex anti $\omega - Q$ −fuzzy sets of rings S_1 and S_2, respectively. If $A^\omega \times B^\omega$ is a complex anti $\omega - Q$ −fuzzy subring of $S_1 \times S_2$, then the conditions hold are,

(i) $\eta_{A^\omega}(0,q) \leq \eta_{B^\omega}(n,q)$ and $\varphi_{A^\omega}(0,q) \leq \varphi_{B^\omega}(n,q)$, for all $n \in S_2$ and $q \in Q$

(ii) $\eta_{B^\omega}(0',q) \leq \eta_{A^\omega}(m,q)$ and $\varphi_{B^\omega}(0') \leq \varphi_{A^\omega}(m)$, for all $m \in S_1$ and $q \in Q$

Where 0 and $0'$ are identities of S_1 and S_2 respectively.

Proof: Let $A^\omega \times B^\omega$ be a complex anti $\omega - Q$ −fuzzy subring of $S_1 \times S_2$. Suppose the two conditions (1) and (2) do not hold. Then $\exists\ m \in S_1 \&\ n \in S_2\ \&\ q \in Q$:

(i) $\eta_{A^\omega}(0,q) \leq \eta_{B^\omega}(n,q)$ and $\varphi_{A^\omega}(0,q) \leq \varphi_{B^\omega}(n,q)$

(ii) $\eta_{B^\omega}(0',q) \leq \eta_{A^\omega}(m,q)$ and $\varphi_{B^\omega}(0',q) \leq \varphi_{A^\omega}(m,q)$

Consider $\theta_{A^\omega \times B^\omega}\left((m,n),q\right) = \max\{\eta_{A^\omega}(m,q),\eta_{B^\omega}(n,q)\}\, e^{i\max\{\varphi_{A^\omega}(m,q),\varphi_{B^\omega}(n,q)\}}$

$\leq \max\{\eta_{A^\omega}(0,q),\eta_{B^\omega}(0',q)\}e^{i\max\{\varphi_{A^\omega}(0,q),\varphi_{B^\omega}(0',q)\}} = \theta_{A^\omega \times B^\omega}\left((0,0'),q\right)$

But $A^\omega \times B^\omega$ is complex anti $\omega - Q$ −fuzzy subring. The fowling two conditions is must be hold.

(i) $\eta_{A^\omega}(0,q) \leq \eta_{B^\omega}(n,q)$ and $\varphi_{A^\omega}(0,q) \leq \varphi_{B^\omega}(n,q)$, for all $n \in S_2$ and $q \in Q$

(ii) $\eta_{B^\omega}(0',q) \leq \eta_{A^\omega}(m,q)$ and $\varphi_{B^\omega}(0',q) \leq \varphi_{A^\omega}(m,q)$, for all $m \in S_1$ and $q \in Q$

Theorem 4.9:

Let $A^\omega and\ B^\omega$complex anti $\omega - Q$ −fuzzy subrings of S_1 and S_2 such that $\eta_{B^\omega}(0',q) \leq \eta_{A^\omega}(m,q)$ and $\varphi_{B^\omega}(0',q) \leq \varphi_{A^\omega}(m,q)$ for all $m \in S_1$ and $0'$ is identity of S_2 and $q \in Q$. If $A^\omega \times B^\omega$ is anti $\omega - Q$-fuzzy subgroup of $S_1 \times S_2$, then A^ω is complex anti $\omega - Q$ −fuzzy subring of S_1.

Proof:

Let $and\ B^\omega$ be two complex anti $\omega - Q$ −fuzzy subrings of S_1 and S_2. Then $(m,0'),(x,0') \in S_1 \times S_2$. By given condition $\eta_{B^\omega}(0',q) \leq \eta_{A^\omega}(m,q)$ and $\varphi_{B^\omega}(0',q) \leq \varphi_{A^\omega}(m,q)$, for all $m,x \in S_1$.

Consider

$\theta_{A^\omega}(m-x,q) = \eta_{A^\omega}(m-x,q)e^{i\varphi_{A^\omega}(m-x,q)}$

$= \max\left\{\eta_{A^\omega}(m-x,q)e^{i\varphi_{A^\omega}(m-x,q)},\eta_{B^\omega}(0'-0',q)e^{i\varphi_{B^\omega}(0'-0',q)}\right\}$

$= \left\{\eta_{A^\omega \times B^\omega}\left((m,0')-(x,0'),q\right)\right\}e^{i\max\left\{\varphi_{A^\omega \times B^\omega}\left((m,0')-(x,0'),q\right)\right\}}$

$\leq \max\left\{\eta_{A^\omega \times B^\omega}\left((m,0'),q\right),\eta_{A^\omega \times B^\omega}\left((x,0'),q\right)\right\}e^{i\max\left\{\varphi_{A^\omega \times B^\omega}\left((m,0'),q\right),\varphi_{A^\omega \times B^\omega}\left((x,0'),q\right)\right\}}$

$=$

$\max\{\max\{\eta_{A^\omega}(m,q),\eta_{B^\omega}(0',q)\},\max\{\eta_{A^\omega}(x,q),\eta_{B^\omega}(0',q)\}\}\, e^{i\max\{\max\{\varphi_{A^\omega}(m,q),\varphi_{B^\omega}(0',q)\},\max\{\varphi_{A^\omega}(x,q),\varphi_{B^\omega}(0',q)\}\}}$

$=$

$\max\{\max\{\eta_{A^\omega}(m,q),\eta_{A^\omega}(m,q)\},\max\{\eta_{A^\omega}(x,q),\eta_{A^\omega}(x,q)\}\}\, e^{i\max\{\max\{\eta_{A^\omega}(m,q),\eta_{A^\omega}(m,q)\},\max\{\eta_{A^\omega}(x,q),\eta_{A^\omega}(x,q)\}\}}$

$= \max\{\eta_{A^\omega}(m,q),\eta_{A^\omega}(x,q)\}\, e^{i\max\{\varphi_{A^\omega}(m,q),\varphi_{A^\omega}(x,q)\}}$

$= \min\{\theta_{A^\omega}(m,q),\theta_{A^\omega}(x,q)\}$

Thus, $\theta_{A^\omega}(m-x,q) \leq \max\{\theta_{A^\omega}(m,q),\theta_{A^\omega}(x,q)\}$

Also, $\theta_{A^\omega}(mx,q) = \eta_{A^\omega}(mx,q)e^{i\varphi_A(mx,q)}$

$= \max\left\{\eta_{A^\omega}(mx,q)e^{i\varphi_A(mx,q)},\eta_{B^\omega}(0'0',q)e^{i\varphi_{B^\omega}(0'0',q)}\right\}$

$= \left\{\eta_{A^\omega \times B^\omega}\left((m,0')(x,0'),q\right)\right\}e^{i\max\left\{\varphi_{A^\omega \times B^\omega}\left((m,0')(x,0'),q\right)\right\}}$

$\leq \max\left\{\eta_{A^\omega \times B^\omega}\left((m,0'),q\right),\eta_{A^\omega \times B^\omega}\left((x,0'),q\right)\right\}e^{i\max\left\{\varphi_{A^\omega \times B^\omega}\left((m,0'),q\right),\varphi_{A^\omega \times B^\omega}\left((x,0'),q\right)\right\}}$

$=$

$\max\{\max\{\eta_{A^\omega}(m,q),\eta_{B^\omega}(0',q)\},\max\{\eta_{A^\omega}(x,q),\eta_{B^\omega}(0',q)\}\}\, e^{i\max\{\max\{\varphi_{A^\omega}(m,q),\varphi_B(0',q)\},\max\{\varphi_{A^\omega}(x,q),\varphi_{B^\omega}(0',q)\}\}}$

$=$

$\max\{\max\{\eta_{A^\omega}(m,q),\eta_{A^\omega}(m,q)\},\max\{\eta_{A^\omega}(x,q),\eta_{A^\omega}(x,q)\}\}\, e^{i\max\{\max\{\eta_{A^\omega}(m,q),\eta_{A^\omega}(m,q)\},\max\{\eta_{A^\omega}(x,q),\eta_{A^\omega}(x,q)\}\}}$

$= \varphi_{A^\omega}$

$= \max\{\theta_{A^\omega}(m,q),\theta_{A^\omega}(x,q)\}$

Thus, $\theta_{A^\omega}(mx,q) \leq \max\{\theta_{A^\omega}(m,q),\theta_{A^\omega}(x,q)\}$

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