Effects Of Radiation, Chemical Reaction And Viscous And Joule Dissipations On Mhd Nanofluid Flow And Heat Transfer Past A Stretching Sheet With Melting

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Abstract

An investigation is done to analyze the Effects of Radiation, Chemical Reaction and Viscous and Joule Dissipations on MHD Nanofluid Flow and Heat Transfer past a Stretching Sheet with Melting in the presence of a uniform transverse magnetic field applied parallel to the stretching sheet. The governing nonlinear partial differential equations are transformed to a set of nonlinear ordinary differential equations which are then solved numerically using Matlab's inbuilt method bvp4c. The behavior of emerging parameters is presented graphically and discussed for velocity, temperature, and nanoparticles volume fraction. Numerical values of skin friction, rate of heat transfer, and rate of mass transfer at the plate are presented in tabular form, for various values of pertinent flow parameters.

Introduction

As we all know, during last few decadesthere is a rapid advancement in scientific, industrial, engineering and technological processes. Due to all these factors the demand of energy is growing day by day.In the present scenario,renewable energy can only be the source of energy to fulfil the growing energy demand. This is the point of attraction for scientists and engineers to develop devices having the property of cooling or heating very fast. This may lead to energy saving and/or storage in a large amount. Such devices could be more suitable economically as well as environmentally. Solar energy is the easiest and most available source of renewable energy in the universe. Solar thermal collectors are designed to capture a large amount of solar radiations. But, in these collectors, conventional fluids are being used as a heat transfer medium. Mostly these fluids have limited capacity to heat-up and low thermal conductivity which ultimately affect the performance. To overcome the problems of low conductivity, researchers forced to develop fluids with enhanced thermal conductivities. Choi and Eastman [1] initiated in this direction and introduced the term “nano-fluid” to refer fluids with suspended nanoparticles. These fluids are peculiar in nature in context withtheir thermo-physical properties. Going forward in this direction, Choi et al. [2] drawn a conclusion that in addition of a very small (<1% of the total volume) quantity of nanoparticles in the base fluid/conventional fluid there is a significant increment (around 40% to 150%) in the thermal conductivity of the base fluids (conventional fluids). After this pioneer work, many researchers [3–5] carried out their investigations on the flow of nano-fluids to understand different aspects of the problem.

Dissipative effects are one of the important factors in many fluid engineering devices. It has been seen that there is a significant heat transfer to the fluid due to the energy dissipation. Kameswaran et al. [6] investigated the "Hydromagneticnanofluid flow due to a stretching or shrinking sheet with viscous dissipation and chemical reaction effect".To describe the nano-fluid flow, they used the model "nanoparticle volume fraction". Hady et al. [7] studied"Radiation effect on viscous flow of a nanofluid and heat transfer over a nonlinearly stretching sheet". Khan et al. [8] carried out an investigation on"Unsteady MHD free convection boundary-layer flow of a nanofluid along a stretching sheet with thermal radiation and viscous dissipation effects". There are several industrial situations where the surface is convectively heated by external source. This causes a certain change in the surface temperature gradient and affects the temperature of the fluid within the boundary layer. Makinde and Aziz [9] investigated "Boundary layer flow of a nanofluid past a stretching sheet with a convective boundary condition". In their governing transport equation, they included Brownian motion and thermophoresis effects. Mahatha et al. [10] studied"Dissipative Effects in Hydromagnetic Boundary Layer Nanofluid Flow past a Stretching Sheet with Newtonian Heating".

Radiative heat transfer phenomenon has its own significance in the construction industries (for representation of gas turbines, reliable stuff, and many propulsion tools for aircraft, missiles, and space vehicles) [11]. Study of heat and mass transfer on the flow of fluids past a stretchable surface including the effect of chemical reaction is equally important as it plays a significant role in chemical as well as metallurgical engineering, e.g. polymer production, food processing etc. Keeping importance of the above, Chamkha and Aly [12] studied steady flow of a polar fluid past a stretchable surface in porous media. They have considered the Soret and Dufour effects and chemical reaction in their problem. Many researchers [13-15] studied fluid flow problems on heat transfer over stretching/shrinking surfaces.

Accompanied with melting (or solidification) characteristics, the heat transfer has wide utilization in the area of melting of permafrost, solidification of magma, and in the process of silicon wafer [16]. Epstein and Cho [17] discussed the usefulness of melting phenomenon in the laminar flow over flat surface. Gireesha et al. [18] studied the stretched flow of viscous nano-liquid with stagnation point, considering melting heat transfer and inclined magnetic field into the problem. Hayat et al. [19] scrutinized the effect of melting parameter in the flow of a chemically reacting fluid. A comprehensive study of heat transfer phenomenon in nano-fluid flow was made by many researchers [20-22]. Recently, Ibrahim [23] studied the melting effects and heat transfer of a nano-fluid past a stretching sheet.

Though the considerable amount of work has been done in the nano-fluid flow over a stretching surface, still more attention is needed to study the melting heat transfer of a radiative nano-fluid flow past a stretching sheet with chemical reaction. Hence authors were motivated to investigate the effects of radiation and melting heat transfer on MHD nano-fluid flow over a stretchable surface. Objective of the present paper is to present a numerical investigation of the steady flow of a viscous, incompressible, electrically conducting, and heat radiating nanofluid over a stretching sheet, with melting, in the presence of an applied transverse magnetic field under different conditions and configurations. The effects of several physical parameters, such as, magnetic field, heat radiation, melting of the sheet etc. on the fluid velocity, heat and mass transfer aspects of the problem are investigated numerically using Matlab's inbuilt method "bvp4c". The fluid flow model considered in the present paper may find applications in high-viscosity fluid engineering devices which work under the influence of an external magnetic field, in fluid flow problems where the cohesive force between the fluid and surface is comparatively larger than the adhesive forces between fluid particles and in flows taking place at high altitudes, in fluid flow problems where the temperature differences between the ambient fluid and flow field is very high and in situations where the whole flow field is heated by some external means.

Formulation of the Problem and its Solution

Consider a steady MHD two-dimensional boundary layer flow and heat transfer of a viscous, incompressible, electrically conducting, heat radiating and chemically reactingnanofluid past a stretching sheet. Viscous and Joule dissipations have also been taken into consideration. The sheet is melting steadily. The flow is subjected to a constant transverse magnetic field of uniform strength $B=B_0$ which is applied in the positive y-direction, normal to the surface. A coordinate frame has chosen that x-axis is extending along the stretching sheet and y-axis is normal to the sheet (figure 1). Temperature of the surface is T_m, concentration C take constant value C_w . The ambient value attained as y tends to infinity of T and C are denoted by T_∞ and C_∞, respectively, while $T_\infty > T_m$. The velocity of the stretching sheet is $u_w = ax$, where a and b are positive constants. Fig. 2.1 shows the geometry of flow model.

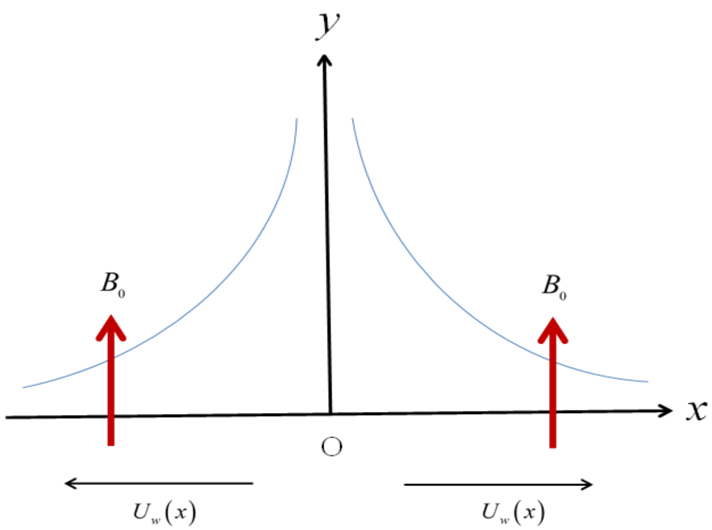

Fig. 1 Geometry of the problem

The governing equations are given by Yacob et al. as:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2}+\frac{\sigma B_0^2}{\rho_f}u \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\tau\left\{D_B\frac{\partial C}{\partial y}\frac{\partial T}{\partial y}+\frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right\}+\frac{\mu}{\rho C_p}\left(\frac{\partial u}{\partial y}\right)^2+\frac{\sigma B_0{}^2}{\rho C_p}u^2-\frac{1}{(\rho C)_f}\frac{\partial q_r}{\partial y} \tag{3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_B\frac{\partial^2 C}{\partial y^2}+\frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial y^2}-k_1\left(C-C_w\right) \tag{4}$$

where $\alpha=\frac{k}{(\rho c)_f}$, $\tau=\frac{(\rho c)_p}{(\rho c)_f}$

The boundary conditions are:

$$\left.\begin{array}{ll} u=u_w=ax,\ v=0,\ T=T_m,\ C=C_w & \text{at } y=0 \\ u=v=0,\ T\to T_\infty,\ C\to C_w & \text{at } y\to\infty \\ \alpha\left(\frac{\partial T}{\partial y}\right)_{y=0}=\rho\left[\lambda+C_s\left(T_m-T_0\right)\right]\upsilon(x,0) & \end{array}\right\} \tag{5}$$

where u and v are the components of velocity along the x and y axes, respectively. Furthermore, $\upsilon,\ \sigma,\ \rho_f,\ \rho_p,\ \alpha,\ k,\ (\rho c)_f,\ (\rho c)_p,\ \lambda$ and C_s are respectively the kinematic viscosity coefficient, electric conductivity, density of base fluid, density of nanoparticle, thermal diffusivity, thermal conductivity, heat capacity of the base fluid, heat capacity of the nanoparticle material, latent heat of the fluid, and heat capacity of the solid surfacewhere k is the thermal conductivity of the fluid, λ is the latent heat of the fluid and C_s is the heat capacity of the solid surface.

The similarity and dimensionless variables are introduced as follow:

$$\left.\begin{array}{l} u=\frac{\partial\psi}{\partial y},v=-\frac{\partial\psi}{\partial x},\eta=y\sqrt{\frac{a}{\upsilon}},\psi=\sqrt{a\upsilon}xf(\eta) \\ \theta(\eta)=\frac{T-T_m}{T_\infty-T_m},\ \phi(\eta)=\frac{C-C_w}{C_\infty-C_w} \end{array}\right\} \tag{6}$$

The equation of continuity is satisfied if we choose a stream function $\psi(x, y)$ such that

$$u = \frac{\partial \psi}{\partial y}, v = -\frac{\partial \psi}{\partial x} \tag{7}$$

With the help of above transformations, equation (1) is identicallysatisfied, and equations (2), (3) and (4) along withboundary conditions (5) take the following forms:

$$f''' + ff'' - f'^2 + A^2 + Mf' = 0 \tag{8}$$

$$\left(1+4R\right)\theta'' + \Pr\left(f\theta' + Nb\varphi'\theta' + Nt\theta'^2 + Ecf''^2 + MEcf'^2 \right) = 0 \tag{9}$$

$$\phi'' + Lef\phi' + \frac{Nt}{Nb}\theta'' - Kr\phi = 0 \tag{10}$$

The corresponding boundary conditions are:

$$\left.\begin{aligned} & f'\left(0\right)=1,\ B\theta'\left(0\right)+\Pr f\left(0\right)=0,\ \theta\left(0\right)=0,\ \phi\left(0\right)=0,\ \text{at}\ \eta=0, \\ & f'\left(\infty\right)\to 0,\ \theta\left(\infty\right)\to 1,\ \phi\left(\infty\right)\to 1,\ \text{as}\ \eta\to\infty \end{aligned}\right\} \tag{11}$$

where the governing parameters are defined by:

$$\left.\begin{aligned} & M=\frac{\sigma {B_0}^2}{\rho_f a},\ Le=\frac{\upsilon}{D_B},\ \Pr=\frac{\upsilon}{\alpha},\ R=\frac{4\sigma^* {T_\infty}^3}{3\alpha^* k},\ Kr=\frac{k_1\upsilon}{aD_B},\ Nb=\frac{\left(\rho c\right)_p D_B\left(C_\infty - C_w\right)}{\left(\rho c\right)_f \upsilon}, \\ & Nt=\frac{\left(\rho c\right)_p D_B\left(T_\infty - T_m\right)}{\left(\rho c\right)_f \upsilon T_\infty},\ B=\frac{C_f\left(T_\infty - T_m\right)}{\lambda + C_s\left(T_m - T_0\right)},\ Ec=\frac{a^2x^2}{C_p\left(T_\infty - T_m\right)}=\frac{{u_w}^2}{C_p\Delta T} \end{aligned}\right\} \tag{12}$$

where f', θ and ϕ are the non-dimensionless velocity, temperature and concentration respectively. M, Le, $\Pr$, Nb, Nt, R, B, Ec and Kr are respectively,the magnetic parameter, Lewis number, Prandtl number, Brownian diffusion coefficient, thermophoretic diffusion coefficient, heat radiation parameter, melting parameter, Eckert number and chemical reaction rate parameter. Dimensionless melting parameter (B) is the combination of Stefan numbers $\frac{C_f\left(T_\infty - T_m\right)}{\lambda}$ (for liquid phase) and $\frac{C_S\left(T - T_0\right)}{\lambda}$ (for solid phase).

Physical quantities which are of much interest in view of the engineering applications, are local skin friction coefficient C_f,the local Nusselt number $Nu_x$and the local Sherwood number Sh_x . These quantities are defined as,

$$C_f = \frac{\tau_w}{\rho {u_w}^2}, Nu_x = \frac{xq_w}{k\left(T_\infty - T_m\right)}, Sh_x = \frac{xh_m}{D_B\left(C_\infty - C_w\right)} \tag{13}$$

where the wall shears stress τ_w , wall heat flux q_w and wall mass flux h_m are given by

$$\tau_m = \mu \frac{\partial u}{\partial y}, q_w = -k\left(\frac{\partial T}{\partial y}\right)_{y=0}, h_m = -D_B\left(\frac{\partial C}{\partial y}\right)_{y=0} \tag{14}$$

Using similarity variables and the above equations, we obtain

$$C_f\sqrt{\mathrm{Re}_x} = -f''(0), \frac{Nu_x}{\sqrt{\mathrm{Re}_x}} = -\theta'(0), \frac{Sh_x}{\sqrt{\mathrm{Re}_x}} = -\phi'(0) \tag{15}$$

where Re_x, Nu_x, Sh_x are local Reynolds number, local Nusselt number and local Sherwood number, respectively.

Numerical Procedure

The non-linear ordinary differential equations (8) - (10) withboundary conditions (11) have been solved using by the bvp4c routine of Matlab.

Results and Discussion

To analyse the effects of flow parameters viz. magnetic parameter M, Brownian motion parameter Nb, Thermophoresis parameter Nt, Radiation parmeter$R$, Melting parameter B,Chemical reaction rate parameter Kr, Eckert number Ec, the profiles of nanofluid velocity, nanofluid temperature and nanoparticles concentration are depicted graphically in figures 2 to 18 while the values of skin friction coefficient $-f''(0)$, local Nusselt number $-\theta'(0)$ and local Sherwood number $-\phi'(0)$ are tabulated in table 1.

Figures 2 to 7 represent effects of magnetic field, thermal Brownian diffusion, thermophoretic diffusion, radiation, melting of the sheet and chemical reaction on the nanofluid velocity. It is observed from figures 2 to 7 that Brownian diffusion, thermophoretic diffusion, chemical reaction and melting of the sheet have the tendency to enhance the nanofluid velocity while magnetic field and radiation have the reverse effect on it.

Figures 8 to 14 demonstrate the effects of various flow parameters of the fluid temperature. It is revealed from fig. 8 to 14 that the magnetic field, radiation and melting of the sheet are the cause for decrease in fluid temperature whereas the thermophoretic diffusion is the cause for increase in fluid temperature. The Brownian diffusion and the viscous deceptionhave the tendency to enhance fluid temperature at the vicinity of the plate. In the boundary layer region, chemical reaction has the tendency to reduce the fluid temperature.

Figures 15 to 21 show the effects of various parameters on fluid concentration. It is concluded from figures 15 to 21 that magnetic field, thermophoretic diffusion, chemical reaction, melting of the sheet and viscous dissipation have the tendency to reduce fluid concentration whereas Brownian diffusion and radiation have the reverse effect on it.

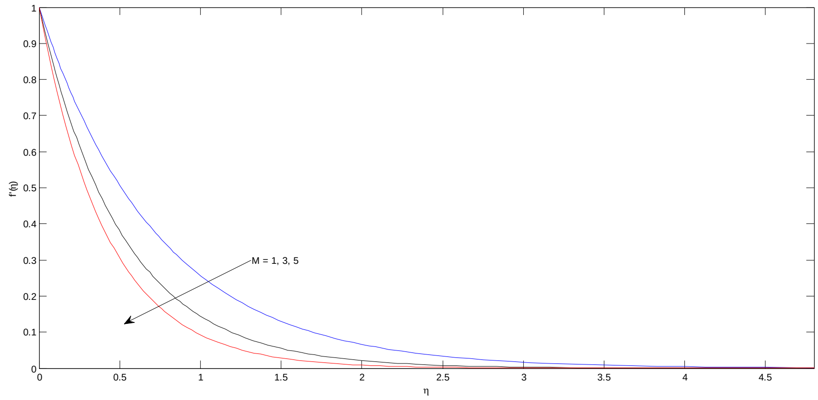

Figure 2: Velocity profiles for *M*

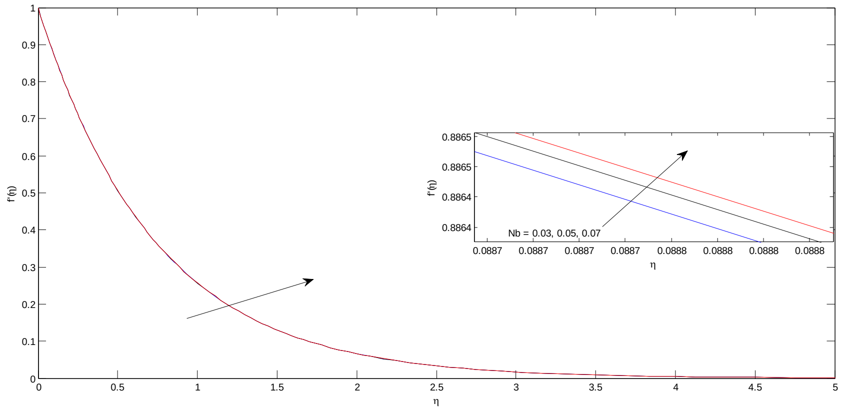

Figure 3: velocity profiles for *Nb*

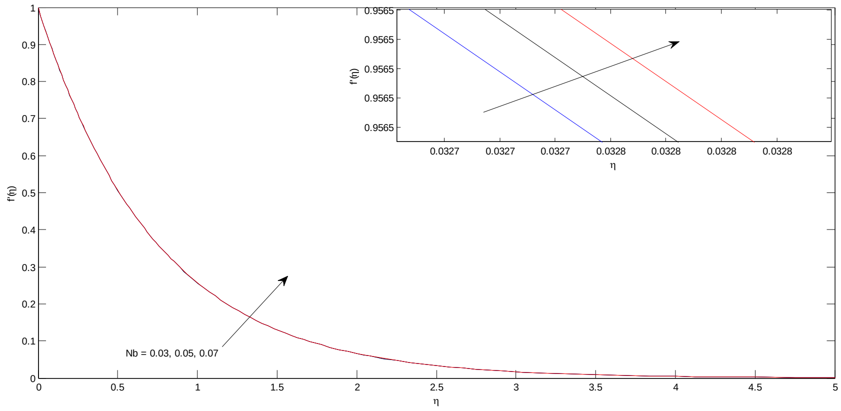

Figure 4: velocity profile for *Nt*

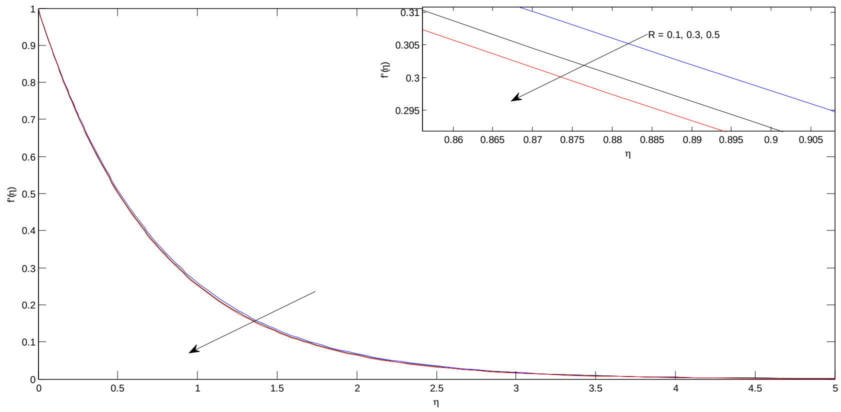

Figure 5: Velocity profile for *R*

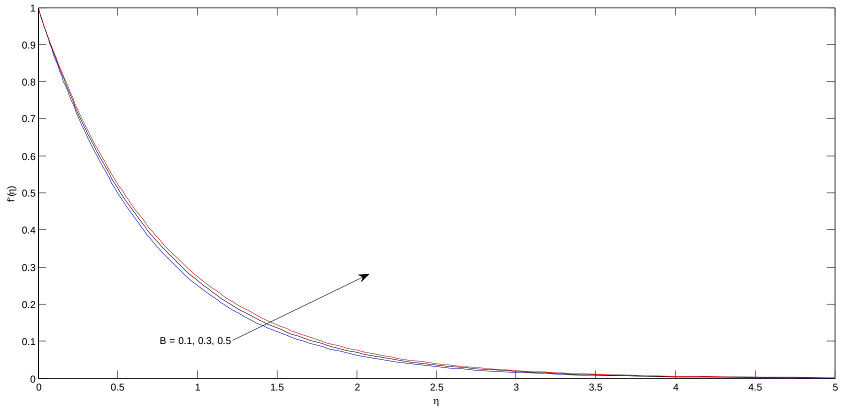

Figure 6: Velocity profiles for*B*

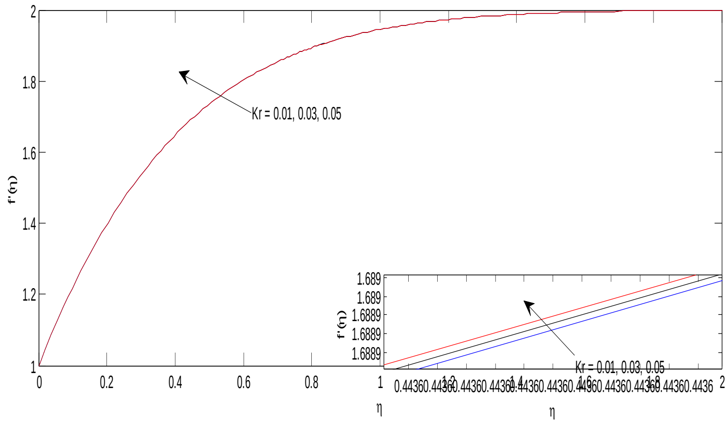

Figure 7: Velocity profiles for *Kr*

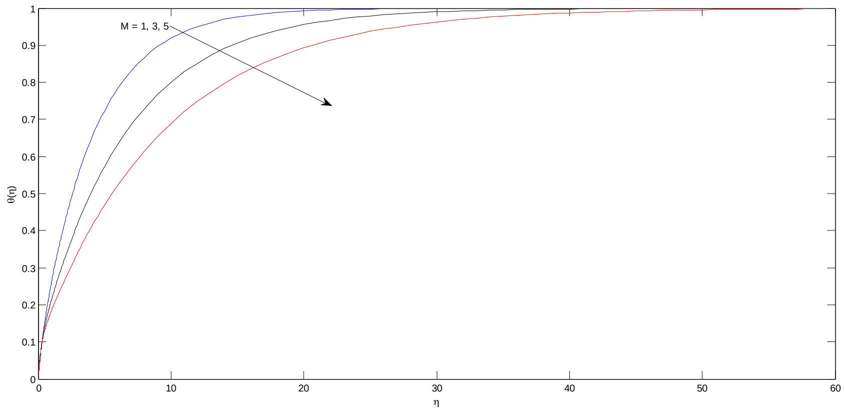

Figure 8: Temperature profiles for *M*

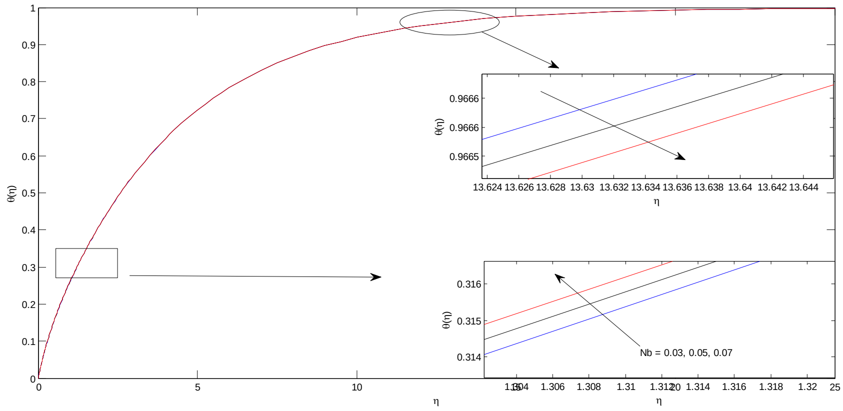

Figure 9: Temperature profiles for *Nb*

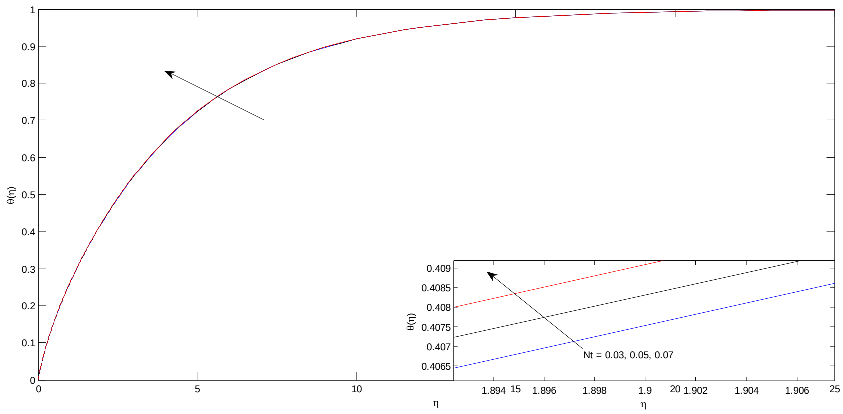

Figure 10: Temperature profiles for *Nt*

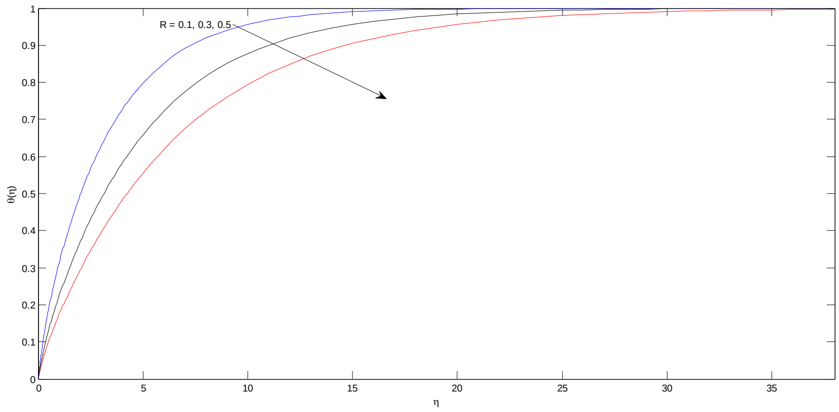

Figure 11: Temperature profiles for *R*

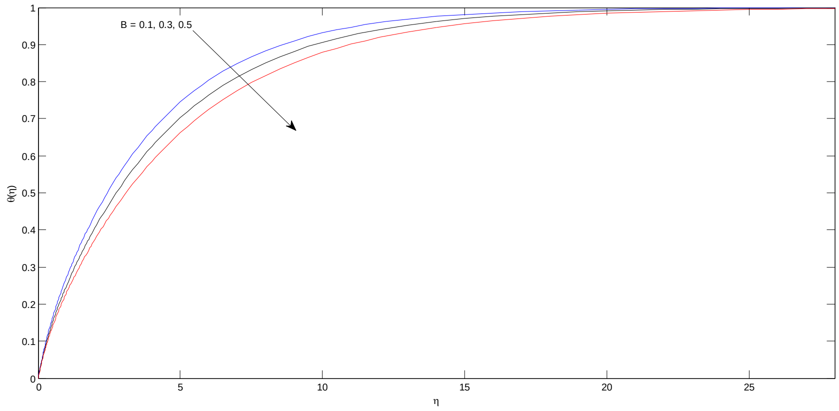

Figure 12: Temperature profiles for *B*

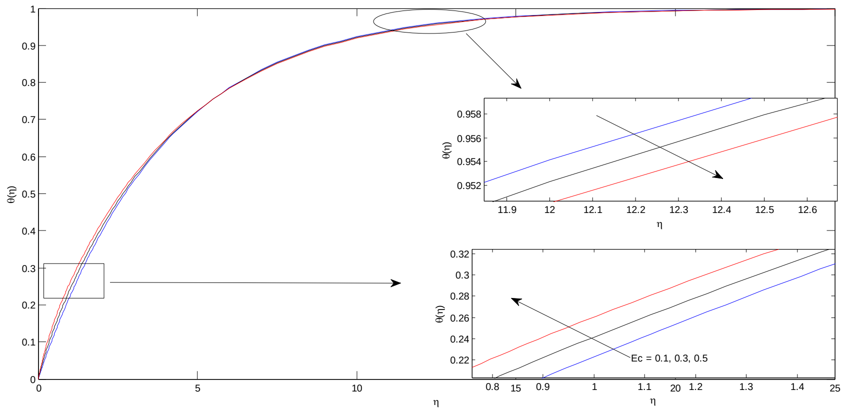

Figure 13: Temperature profiles for *Ec*

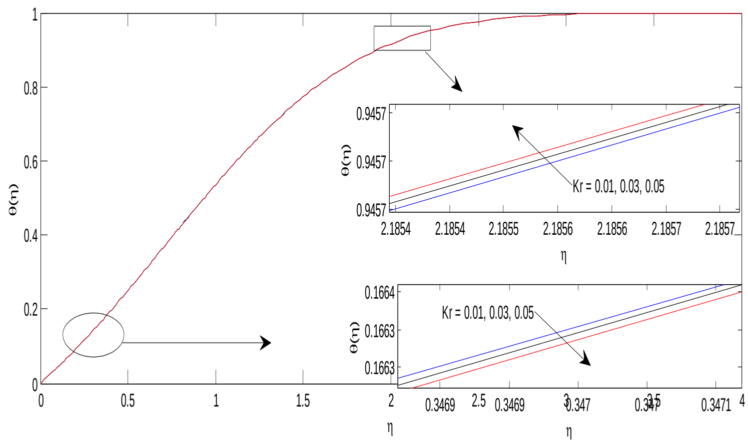

Figure 14: Temperature profiles for *Kr*

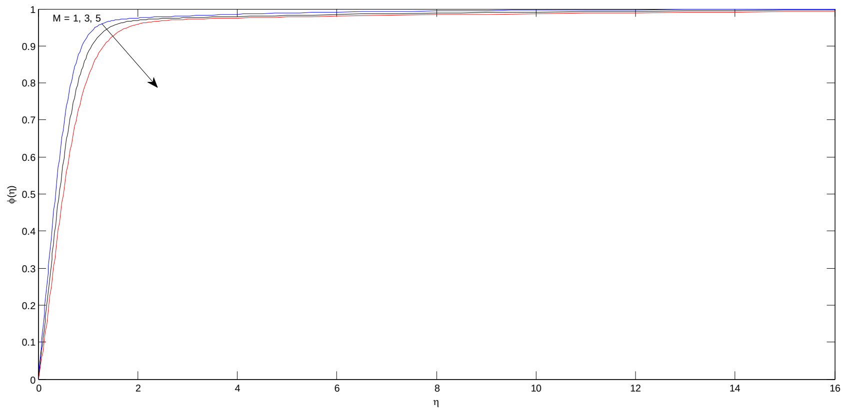

Figure 15: Concentration profiles for *M*

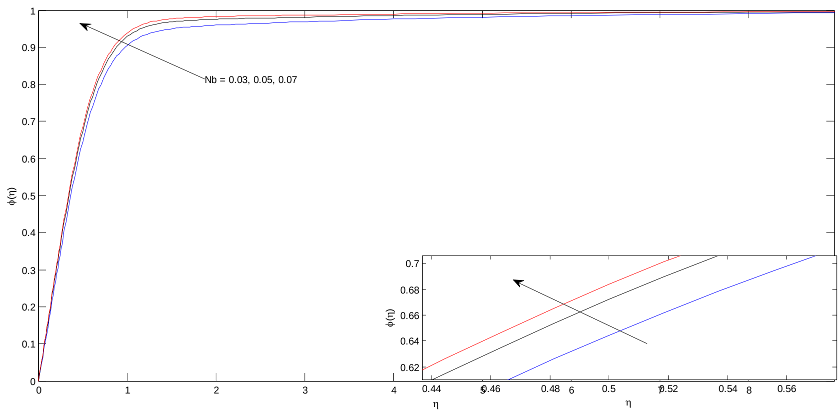

Figure 16: Concentration profiles for *Nb*

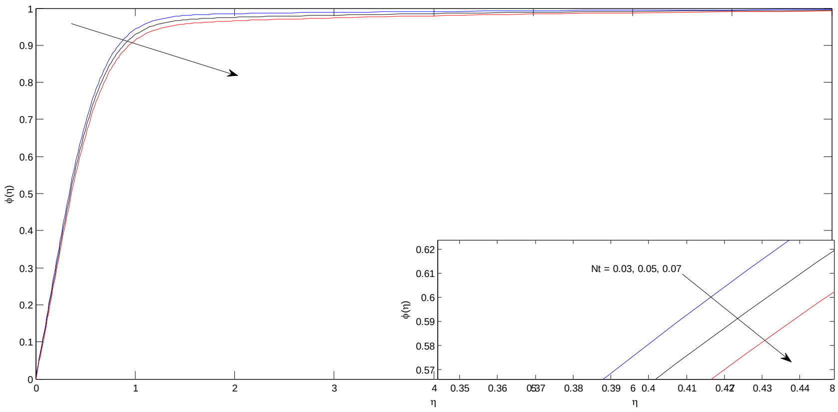

Figure 17: Concentration profiles for *Nt*

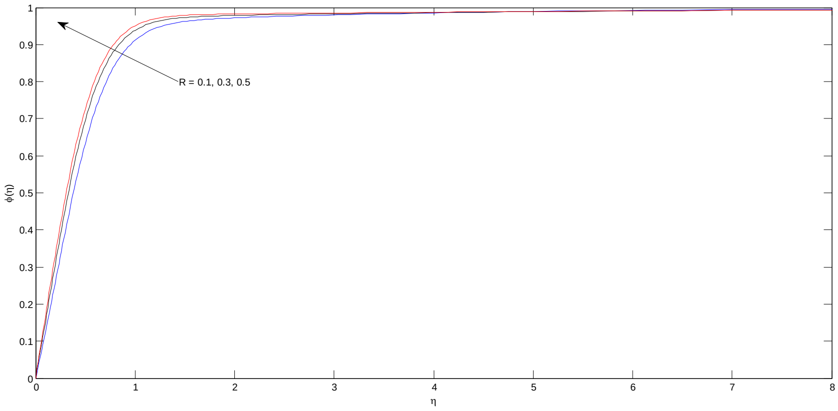

Figure 18: Concentration profiles for *R*

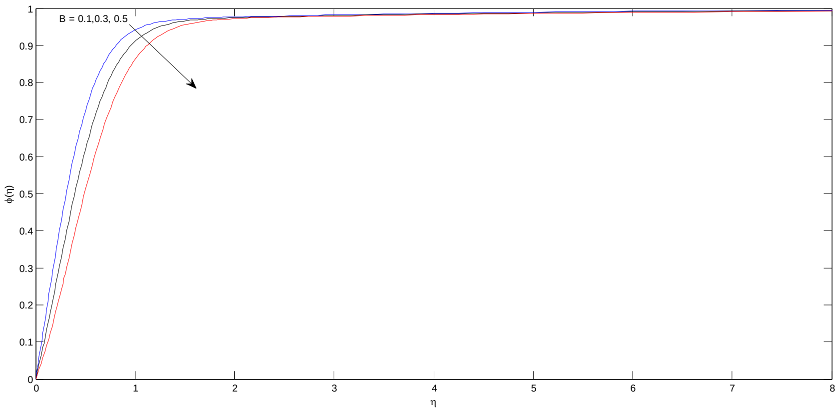

Figure 19: Concentration profiles for *B*

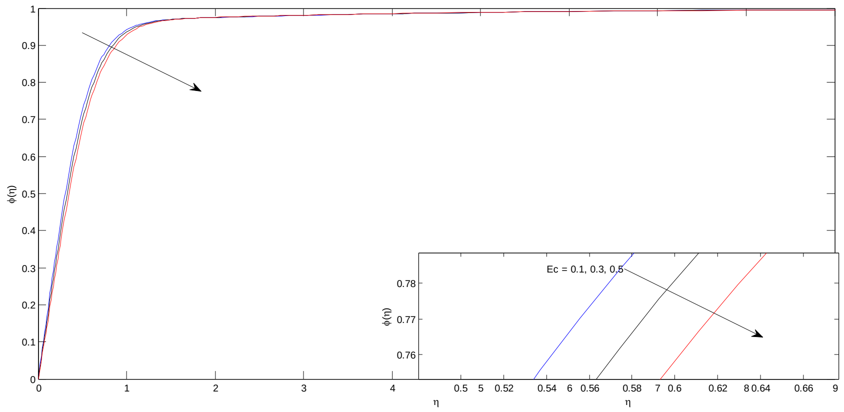

Figure 20: Concentration profiles for *Ec*

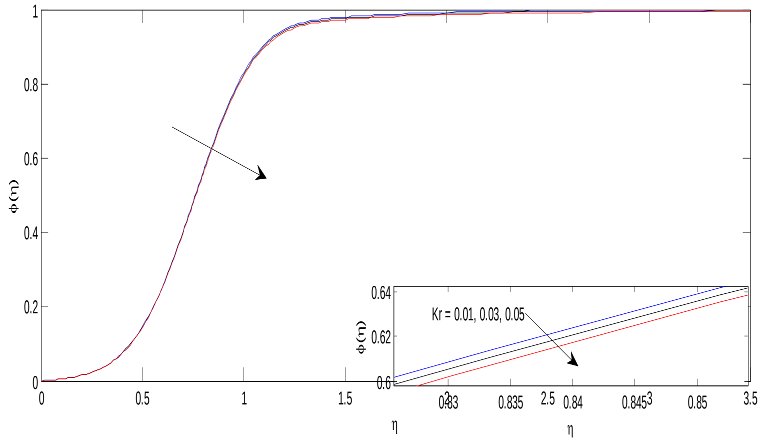

Figure 21: Concentration profiles for *Kr*

Table 1 represents the effects of various parameters in the coefficients of skin friction, Nusselt number and Sherwood number. It is clearly seen from the table that the coefficient of skin friction is an increasing function of *M, Pr* and *R* whereas it is a decreasing function of *Nb, Nt, Le, B*,and*Ec.* With the increase in *M, Pr, Nb, Nt, Le* and *Ec* the Nusselt number is getting increased while it is getting decreased with the increase in R and B. the Sherwood number increases on increasing *Nb, R* and *Le* while it is decreases on increasing *M, Pr, Nt,B* and *Ec*. This implies that magnetic field and radiation induces the coefficient of skin friction, while thermal diffusion, Brownian diffusion, thermophoretic diffusion, Lewis number, melting of the sheet and viscous deception have the reverse effect on it. Magnetic field, Brownian diffusion, thermophoretic diffusion, Lewis number and viscous deception are the cause for the enhancement in rate of heat transfer whereas thermal diffusion and melting of the sheet are the cause for the decrease in rate of heat transfer. Thermal diffusion, Brownian diffusion, radiation and Lewis number induces rate if mass transfer whereas magnetic field, thermophoratic diffusion, melting of the sheet and viscous deception have the tendency to reduce rate of mass transfer.

Table 1 Effects of various parameters on coefficient of skin friction, Nusselt number and Sherwood numbers

M	Pr	Nb	Nt	R	Le	B	Ec	$C_f\sqrt{\text{Re}_x}$	$\frac{Nu_x}{\sqrt{\text{Re}_x}}$	$\frac{Sh_x}{\sqrt{\text{Re}_x}}$
1	0.71	0.05	0.05	0.2	10	0.2	0.5	-1.358128	0.40643	1.33482
3								-1.934117	0.475739	0.974132
5								-2.375084	0.536556	0.679477
	0.71							-1.358128	0.40643	1.33482
	1							-1.359791	0.555114	1.280311
	1.5							-1.362122	0.796312	1.179926
		0.03						-1.358394	0.40446	1.219029
		0.05						-1.358128	0.40643	1.33482
		0.07						-1.35786	0.40841	1.382647
			0.03					-1.358242	0.405587	1.408849
			0.05					-1.358128	0.40643	1.33482
			0.07					-1.358013	0.407277	1.260542
				0.1				-1.345601	0.499571	1.15201
				0.3				-1.366773	0.34267	1.464505

				0.5				-1.377939	0.260942	1.635839
					3			-1.358279	0.405312	0.59677
					5			-1.358209	0.405833	0.89092
					10			-1.358128	0.40643	1.33482
						0.1		-1.384681	0.42383	1.636709
						0.3		-1.334092	0.390632	1.080666
						0.5		-1.292166	0.362984	0.690413
							0.1	-1.377897	0.261246	1.661311
							0.3	-1.367946	0.334058	1.494869
							0.5	-1.358128	0.40643	1.33482

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