*International Journal of Modern Agriculture, Volume 9, No.3, 2020*
*ISSN: 2305-7246*

# Study of a Population Dynamical Model

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**Abstract**

The dynamics in growth of population of any species is not interdependent only with other species but also with other factors like environment,resources and pesticides. In this paper we have studied and modelled on the dynamics in growth of population of a single species. The population growth occurred exponentially. Though the Single-species models are meant for only laboratory investigation but, it's schema or model can be utilized to determine or study about impact of other species in growth of any organism.

**Keywords:** Population , species, dynamics

## Introduction

The growth of study on population dynamics of different species with different parameter signifies the concern of research on endangered species, bacterial or viral species along with other species. The population dynamics depends upon different ecological factors like predator-prey ,availability of resources , use of pesticides and its impact on environment, control of genetically engineered species. Not some time but many times, variation or change of any habitants is interrelated with the variation of other species. and the environment.

In this paper the variation of total habitant of a system of a single class is considered. Hilborn and Mangel (1997) faced models with data and discussed many real world case studies of population growth with probabilistic approach. For development of a suitable ecosystem for approaching existence and continue of prevailing of the renewable resources of flora and fauna or whatso ever. It is of natural wish for the highest exist able harvest with the less endeavour. In the present scenario, researchers are more interested for including monetary benefit along with the stability of the population model. ."Kot (2001)" has well-defined on yielding models and optimum control theory. "Plant and Mangel (1987)" had studied on pest control theory". The model described that a plain logistic model with the enclosure of a "harvesting weight" and was analysed by "Beddington and May (1977)". Though it is a mostly straightforward one but it gives a numerous motivation and plays vital role which can be applied in more complicated models. Later on Rotenberg consider the same model with quantitively manner.

## Modelling of the problem:

The models consists of single species are the Skelton of multi species models and it plays vital role in laboratory simulation. But it can not be directly applied for simulation of multi species model directly rather it required some modifications according to the impact of different parameter playing role.

Assume that the total inhabitants of the species at any moment t be X (t), and *b* and *d* be the average per heads birth rate and death rate, respectively. For a small time $\Delta t$, the number of births in the population is $b\Delta tX$, and the number of deaths is $d\Delta tX$. Then the population *X* at the moment $t + \Delta t$ and is mentioned as

$X(t + \Delta t) = X(t) + b\Delta t\, X(t) - d\Delta t\, X(t)$

This can be expressed as $\frac{X(t+\Delta t)-X(t)}{\Delta t} = (b-d)X(t)$

$$As\ \Delta t \to 0, \frac{dX}{dt} = (b-d)X \qquad (2.1)$$

"Rate of enhance of population = birth rate – death rate + rate of immigration – rate of emigration" (2.2)

Let's assume the system is closed and thus there is no "immigration or emigration"

So $\frac{dX}{dt} = bX - dX$ is the net population of the species at any moment.

| **Date** | **Mid 17th Century** | **Early 19th Century** | **1918-1927** | **1960** | **1974** | **1987** | **2000** | **2050** | **2100** |
|---|---|---|---|---|---|---|---|---|---|
| inhabitants in billions | 0.5 | 1.0 | 2.0 | 3.0 | 4.0 | 5.0 | 6.3 | 10.0 | 11.2 |

The equation (2.1) represents the simple modelling and it can be made necessary modification depending upon situation. Since the model have not been considered migrations so it is the simplest model and the population growth rate at any time is depending upon the population of that time X. In other words it can be stated as the birth rate and death rate also proportionate with the population X

1. "Continuous Population Models for Single Species"

$$\Longrightarrow \frac{dX}{dt} = bX - dX$$

$$\Rightarrow \frac{dX}{dt} = bX - dX \qquad (2.3)$$

and Then integrating both side we get

$$\Rightarrow log\left(\frac{X}{X_0}\right) = (b-d)t$$

$$X(t) = X_0 e^{(b-d)t} \qquad (2.4)$$

Here b, d are +ve constraints.

The opening inhabitants $X(0) = X_0$. So, if $b > d$ then the inhabitants growth happened exponentially and if $b < d$ then the species vanishes. The approach was stated by Malthus 2 in 1978, is quite impractical. For the entire world population,let us think the previous and future growth calculation since 17th to 21st centuries we find it is less impractical that can be viewed from table-1.Since 1900 it has grown exponentially.

It is hard to build lengthy term, or even reasonably short term guss, except one have adequate suggestion to integrate in the model to convert more consistent forecaster. Though the usual trends can give only a rough idea but it is not correct quantitively..

The world is educated day by day and have adopted different types of contraception and sterilisations, the data says26% of female sterilisation and only 10 % of male along with regional famine and pandemic ,still the world population is growing hurriedly. So it is the time to check the exponential curve growth rate by implementing some actions.

Verhulst (1838, 1845) anticipated "self-limiting" processes that drive when a inhabitants is so big.

$$\Rightarrow \frac{dX}{dt} = rX\left(1 - \frac{X}{k}\right) \quad (2.5)$$

Solving above equation by using separation of variable concepts we get

$$\frac{dX}{\left(1-\frac{X}{k}\right)X} = rdt (2.6)$$

By making partial fractions we get

$$\frac{dX}{k-X} + \frac{dX}{X} = rdt \quad (2.7)$$

Integrating both the sides and rearranging we get

$$\int \frac{dX}{k-X} + \int \frac{dX}{X} = r\int dt \quad (2.8)$$

After substituting initial condition, we get

$$\Rightarrow x(t) = \frac{X_0 k e^{rt}}{k + X_0(e^{rt} - 1)} \quad (2.9)$$

where "r and L" are +ve constraints.

This he known as"logistic growth"in a population. In this model the per heads birth rate is r (1 – X / L); i.e, it reliant on X. The constant L is the "carrying capacity" of the surroundings, that is generally evaluated by the availability nourishing possessions.

There are two states known as "steady states and equilibrium states", named as X = 0 and X = L,i.e, where dX/dt = 0. X = 0 is not stable since linearization about it (that is, X2 is neglected compared with X) gives dX / dt = rX, and so Xraises exponentially from any of a small initial value. The other equilibrium X = L is stable: linearization about it (that is, $(X - L)^2$ is neglected compared with |X–L|) gives d (X–L)/dt = -r (X – L) and so X→L at t →∞. The carrying capacity L decides the size of the stable "steady state population" while r is a evaluate of the rate at which it is reached; i.e, it is a measure of the dynamics.We could take it in the time by a alteration from t to rt. Thus 1 / r is a indicative "timescale"of the reaction of the model to any alter in the inhabitants.

If $X(0) = X_0$ the solution of (2.9) is

$$X(t) = \frac{X_0 L e^{rt}}{[L + X_0(e^{rt} - 1)]} \longrightarrow k \; as \; t \longrightarrow \infty (2.10)$$

and is shown in Figure 1.1. If $X_0<L$,X(t) is increases monotonically to L while if $X_0>L$ it is decreases "monotonically" to L. In the earlier case there is a qualitative modification depending onwhether $X_0>L/2$ or $X_0<L/2$; with $X_0<L/2$ the form has a typically "sigmoid character", which is mainly observed.

In the case where $X_0>L$, this would suggest that the per heads birth rate is negative.

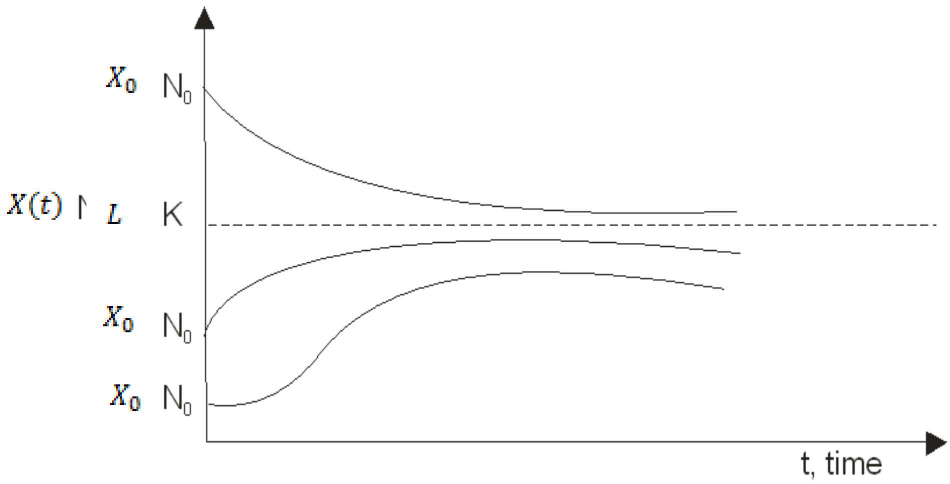
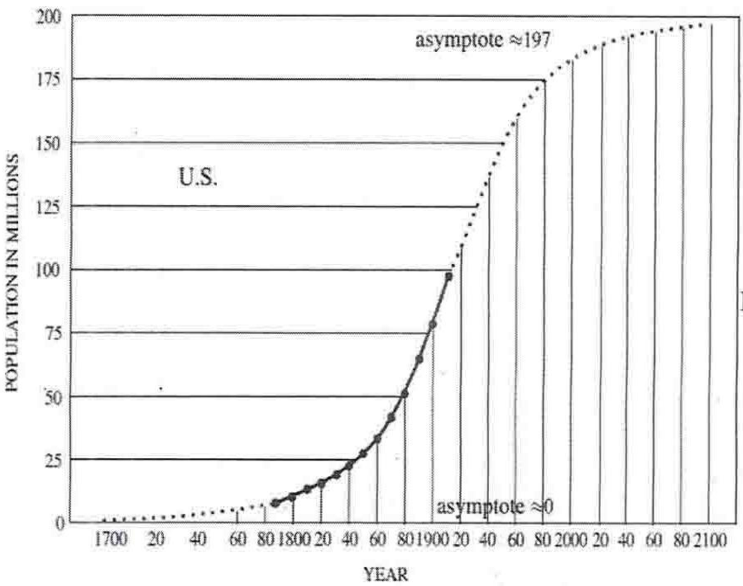


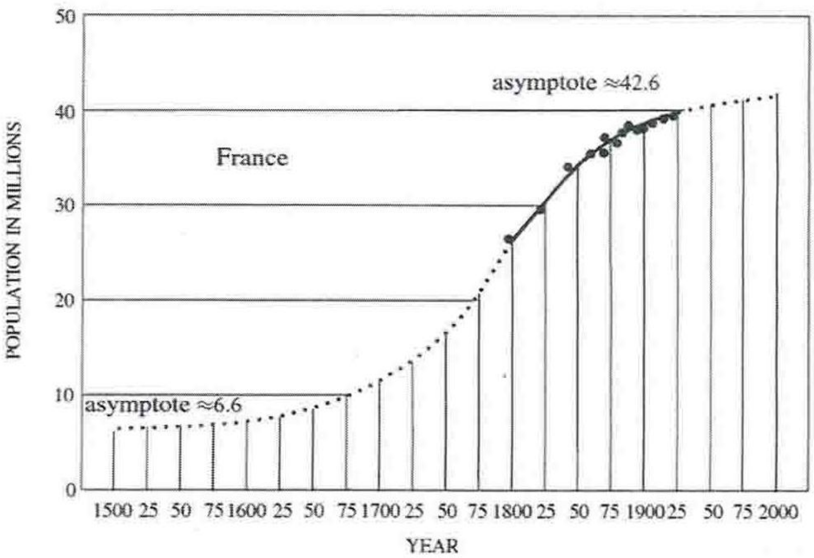

**Fig-2 Logistic population growth.**

Note the qualitative transformation for the two cases $X_0<L/2$ an $X>X_0>L/2$.

The logistic form will happen in a diversity of different contexts right through the book primary because of its algebraic simplicity and it affords a introductory qualitative idea of what can occur with more realistic forms.




(a)
FIGURE
3(a)




(b)

"Logistic population growth" (2.1) use to acceptable the census data for the population of (a) the U.S. and (b) France. The data accomplish the parameters only over a small part of the growth curve.

Generally if we think a "population" to be ruled by where typically g(X) is a nonlinear function of *X* then the "equilibrium solutions"$X^*$ are solution of g(X) = 0 (and are linearly stable to small perturbations if $g'(X^*) < 0$ and stable if $g'(X^*) > 0$.This is reasonable rom linearising about $X^*$ by writing x(t) = X(t) – $X^*$, |x(t)| << 1

We know $x(t) = ce^{\lambda t} \Rightarrow \lambda = e^{-\lambda t}$

then $\frac{dX}{dt} = g(X^* + x) \approx g(X^*) + xg'(X^*) + - - -$ (2.11) which is first order in n(t) gives,

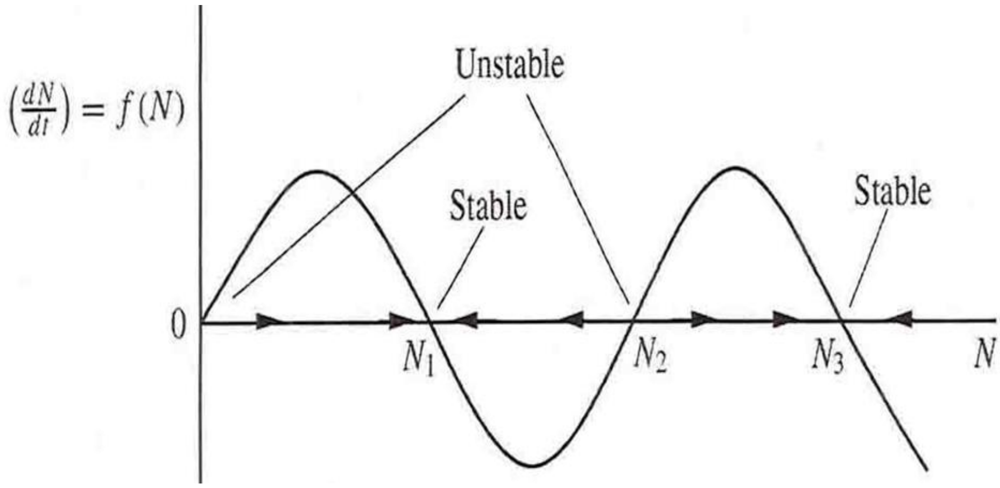
$$\frac{dX}{dt} \approx ng'(X^*) \to x(t)\alpha e^{[g'(X^*)t]} \tag{2.12}$$

So *x*raises or declines accordingly as $g'(X^*) > 0$ or $g'(X^*) < 0$. The "timescale of the response" of the population to a disturbance is of the order of $1/|g'(X^*)|$it is the time to change the initial disturbance by a factor *e*.

There may be a number of"equilibrium, or steady state", population $X^*$ which are solutions of g(X) = 0; it depends on the system g(X) models. Graphically plotting g(X) against X immediately gives the "equilibrium" as the points where it crosses the X-axis. The "gradient"$g'(X)$ at each steady state then determines its linear stability. Such steady states may be unstable to finite instability. Suppose, that g(X) is as shown in Figure 3(a) & 3(b). The gradients $g'(X^*)$atX = 0, X2 are positive so that equilibriums became unstable whereas at X = X1, X3 are steady to small perturbations."stability or instability" are shown by the arrow mark.

If, we perturb the population from its equilibrium X1 so that X will range X2 <X<X3 then X→X3 rather than returning to X1. Ananalogous perturbation from X3 to a value in the range 0 <X<X2 would outcome in X(t) →X1.

Qualitatively there is a "threshold perturbation" underneath that the "steady states" are all the time stable, and this threshold depends on the nonlinear from off(X).




For X1, for illustration, the necessary threshold perturbation is X2 – X1.

Population dynamics model$\frac{dX}{dt} = g(x)$ with numerous steady states. The gradient $g'(X)$at the steady state, i.e, The point g(X) = 0, indicates the "linear stability".

**Conclusion:**

Though the single species model has less realistic in nature and can be use for laboratory purpose still opens the idea of incorporating other terms of influences upon the model. The stability of the model is analysed.One conclusion can be drawn from the above modelling is that “a constant effort rather than a constant yield harvesting strategy is less potentially disastrous”.

**References**

1. Tepperman, L. Malthus , the social limits to growth. Soc Indic Res 9, 347–368 (1981).
2. Ray Hilborn , Marc Mangel (1997) The Ecological Detective: Confronting Models with Data (MPB-28) ISBN: 978-1-4008-4731-0
3. Bacaër N. (2011) Verhulst and the logistic equation (1838). In: A Short History of Mathematical Population Dynamics. Springer, London.
4. Richard E Plant, Marc Mangel Modeling and Simulation in Agricultural Pest Management (1987) SIAM Review Vol-29. Issue-2 , pp-235-261
5. Beddington, J.R. and May, R.M. (1977) Harvesting Natural Populations in a Randomly Fluctuating Environment. Science, 197, 463-465.
6. Michal Rotenberg,Rimona Margalit,Smadar Cohen,Thermodynamics Of Porphyrin Binding To Serum Albumin: Effects Of Temperature, Of Porphyrin Species And Of Albumin-Carried Fatty Acids
7. F.Brauer D.Sanchez (1975) Constant rate population harvesting equilibrium.
8. Raymond Pearl (1925), The biology of Population growth, New york Academy of Medicines.