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A Study On Bipolar Valued Antifuzzy Km Ideal On K Algebras

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Abstract
In this article, Fuzzy KM-ideal on K algebras, Anti Fuzzy KM-ideal, bipolar valued fuzzy set are discussed and bipolar valued anti fuzzy KM-ideal on K-Algebras introduced and Properties are discussed.

Keywords:K-algebras, KM-ideals, Fuzzy KM-ideals, Anti Fuzzy KM-ideals, Cartesian product, Fuzzy relations, Strongest fuzzy relations, Bipolar anti fuzzy KM Ideals on K algebras.

1.Introduction

R.Biswas [1] introduced the concept of Anti fuzzy subgroup of a group. Dar and Akram is introduced K-Algebra $(G,\odot.e)$ [2]. After the introduction of fuzzy sets by Zadeh [3], the fuzzy set theory developed by Zadeh himself and others in many directions and found applications in various areas of sciences. Akram et al introduced the notions of sub algebras and fuzzy (maximal) ideals of K-algebras in [4,5]. Lee [6] introduced the operation in bipolar-valued fuzzy sets. In this Fuzzy ideals of K Algebras are introduced and their properties are verified.The extension of Fuzzy KM an ideal on K-algebras is also hosted and verified [7]. In this article, we discussed about the Bipolar-valued anti fuzzy KM-ideals on K-algebras are introduced and studied their properties with Cartesian product of anti fuzzy KM-ideals and strongest fuzzy relation in detail.

2. Preliminaries

In this section we recap some basic aspects that are necessary for this article:

Definition 2.1.

Let $(G, \cdot, e)$ be a group with the identity e such that $x^2 \neq e$ for some $x(\neq e) \in G$.

A K-algebra built on G (briefly, K-algebra) is a structure $K = (G, ., \odot, e)$ where "$\odot$" is a binary operation on G which is induced from the operation "$\cdot$", that satisfies the following:

(k1) $(\forall a, x, y \in G)\ ((a \odot x) \odot (a \odot y) = (a \odot (y^{-1} \odot x^{-1})) \odot a)$,

(k2) $(\forall a, x \in G)\ (a \odot (a \odot x) = (a \odot x^{-1}) \odot a)$,

(k3) $(\forall a \in G)\ (a \odot a = e)$,

(k4) $(\forall a \in G)\ (a \odot e = a)$,

(k5) $(\forall a \in G)\ (e \odot a = a^{-1})$.

If G is abelian, then conditions (k1) and (k2) are replaced by:

(k1') $(\forall a, x, y \in G)\ ((a \odot x) \odot (a \odot y) = y \odot x)$,

(k2') $(\forall a, x \in G)\ (a \odot (a \odot x) = x)$,

respectively.

A nonempty subset H of a K-algebra K is called a subalgebra of K if it satisfies:

• (∀a, b ∈ H) (a ⊙ b ∈ H).

Note that every subalgebra of a K-algebra K contains the identity e of the group (G, ·).

A mapping f : K1 → K2 of K-algebras is called a homomorphism if f(x ⊙ y) = f(x) ⊙ f(y) for all x, y ∈ K1. Note that if f is a homomorphism, then f(e) = e. A nonempty subset I of a K-algebra K is called an ideal of K if it satisfies:

(i) e ∈ I,

(ii) (∀x, y ∈ G) (x ⊙y ∈ I, y ⊙ (y ⊙ x) ∈ I ⇒ x ∈ I).

Let μ be a fuzzy set on G,

i.e., a map μ : G → [0, 1]. A fuzzy set μ in a K-algebra K is called a fuzzy subalgebra of K if it satisfies:

• (∀x, y ∈ G) (μ(x ⊙ y) ≥ min{μ(x), μ(y)}).

Every fuzzy subalgebra μ of a K-algebra K satisfies the following inequality:

(∀x ∈ G) (μ(e) ≥ μ(x)).

Definition 2.2. A fuzzy set μ in a K-algebra is called a fuzzy KM ideal of K if it satisfies:

(i) (∀x ∈ G) (μ(e) ≥ μ(x)),

(ii) (∀x, y ∈ G) (μ(y) ≥ min{μ(y ⊙ x), μ(x ⊙ (x ⊙ y))}).

Definition 2.3. A fuzzy set μ in a K-algebra is called a anti fuzzy KM ideal of K if it satisfies:

(i) (∀x ∈ G) (μ(e) ≤ μ(x)),

(ii) (∀x, y ∈ G) (μ(y) ≤ max{μ(y ⊙ x), μ(x ⊙ (x ⊙ y))}).

Definition 2.4. A bipolar valued fuzzy set $(B_iFS)\mu$ in P is defined as an object of the form μ = {<p, μ^+(p), μ^-(p) >/ p∈P}, where μ^+: P→ [0, 1] and μ^-: P→ [–1, 0]. The positive membership degree μ^+(p) denotes the satisfaction degree of an element p to the property corresponding to a bipolar valued fuzzy set μ and the negative membership degree μ^-(p) denotes the satisfaction degree of an element p to some implicit counter-property corresponding to a bipolar valued fuzzy set μ.

Definition 2.5. A bipolar valued fuzzy set $(B_iFS)\mu$ in K-Algebra P is said to be a bipolar valued fuzzy KM-ideal (B_iFI_{km}) of P if for every p, q ϵ P

1. $\mu_B^+(0) \geq \mu_B^+(p)$ and $\mu_B^+(0) \leq \mu_B^+(p)$
2. $\mu_B^+(q) \geq \min\{\mu_B^+(q \odot p), \mu_B^+(p \odot (p \odot q)\}$ and
   $\mu_B^-(q) \leq \max\{\mu_B^-(q \odot p), \mu_B^-(p \odot (p \odot q)\}$

Definition 2.6. A bipolar valued fuzzy set $(B_iFS)\mu$ in K-Algebra P is said to be a bipolar valued anti fuzzy KM-ideal (B_iAFI_{km}) of P if for every p, q ϵ P

1. $\mu_B^+(0) \leq \mu_B^+(p)$ and $\mu_B^+(0) \geq \mu_B^+(p)$
2. $\mu_B^+(q) \leq \max\{\mu_B^+(q \odot p), \mu_B^+(p \odot (p \odot q)\}$ and
   $\mu_B^-(q) \geq \min\{\mu_B^-(q \odot p), \mu_B^-(p \odot (p \odot q)\}$

Theorem 2.7. The intersection of any two $B_iAFI_{km}s$ of P is also a B_iAFI_{km}.

Proof.
Let μ = {< p, μ^+(p), μ^-(p) >/ p∈P} and δ = {< p, δ⁺(p), δ⁻(p) >/ p∈P}
Let W = μ ∩δ and W={ < p, W^+(p), W^-(p) >/ p∈P }

Then $W_B^+(0) = \max\{\mu_B^+(0), \delta_B^+(0)\}$
$\leq \max\{\mu_B^+(p), \delta_B^+(p)\}$
$= W_B^+(p)$

And $W_B^-(0) = \min\{\mu_B^-(0), \delta_B^-(0)\}$
$\geq \min\{\mu_B^-(p), \delta_B^-(p)\}$
$= W_B^-(p)$

Also $W_B^+(q) = \max\{\mu_B^+(q), \delta_B^+(q)\}$
$\leq \max\{\max\{\mu_B^+(q \odot p), \mu_B^+(p \odot (p \odot q)\}\{\delta_B^+(q \odot p), \delta_B^+(p \odot (p \odot q)\}$
$= \max\{\max\{\mu_B^+(q \odot p), \delta_B^+(q \odot p)\}, \max\{\mu_B^+(p \odot (p \odot q), \delta_B^+(p \odot (p \odot q)\}\}$
$= \max\{\{\mu_B^+(q \odot p), \mu_B^+(p \odot (p \odot q))\}$

And also $\mu_B^-(q) = \min\{\mu_B^-(q), \delta_B^-(q)\}$
$\leq \min\{\min\{\mu_B^-(q \odot p), \mu_B^-(p \odot (p \odot q)\}\{\delta_B^-(q \odot p), \delta_B^-(p \odot (p \odot q)\}$
$= \min\{\min\{\mu_B^-(q \odot p), \delta_B^-(q \odot p)\}, \min\{\mu_B^-(p \odot (p \odot q), \delta_B^-(p \odot (p \odot q)\}\}$
$= \min\{\{\mu_B^+(q \odot p), \mu_B^+(p \odot (p \odot q))\}$

Hence W = $\mu \cap \delta$ is also a B_iAFI_{km}.

3.Cartesian Product

Definition 3.1. Let μ and δ be the B_iFSs in P and Q respectively. The cartesian product μ x δ: Px Q → [0,1] is defined by (μx δ)={(p,q), $(\mu \times \delta)_B^+(p,q)$, $(\mu \times \delta)_B^-(p,q)$ / ∀ p ∈ P and ∀q∈Q } where $(\mu \times \delta)_B^+(p, q) = \min\{\mu_B^+(p), \delta_B^+(q)\}$ and $(\mu \times \delta)_B^-(p, q) = \max\{\mu_B^-(p), \delta_B^-(q)\}$,∀p ∈P, q∈ Q.

Definition 3.2. Let μ and δ be the B_iAFSs in P and Q respectively. The cartesian product μ x δ: Px Q → [0,1] is defined by (μx δ)={(p,q), $(\mu \times \delta)_B^+(p,q)$, $(\mu \times \delta)_B^-(p,q)$ / ∀ p ∈ P and ∀q∈Q } where $(\mu \times \delta)_B^+(p, q) = \max\{\mu_B^+(p), \delta_B^+(q)\}$ and $(\mu \times \delta)_B^-(p, q) = \min\{\mu_B^-(p), \delta_B^-(q)\}$,∀p ∈ P ,q∈ Q.

Definition 3.3. Let μ be the B_iFS in a set P, the strongest bipolar valued fuzzy relation on P, that is the strongest bipolar valued fuzzy relation on μ is J = {⟨ (p, q), $J_B^+(p, q)$, $J_B^-(p, q)$ ⟩/ p, q ∈P}given by $J_B^+(p, q) = \min\{\mu_B^+(p), \mu_B^+(q)\}$ and $J_B^-(p, q) = \min\{\mu_B^-(p), \mu_B^-(q)\}$,∀p, q ∈ P.

Definition 3.4. Let μ be the B_iAFS in a set P, the strongest bipolar valued fuzzy relation on P, that is the strongest bipolar valued anti fuzzy relation on μ is J = {⟨ (p, q), $J_B^+(p, q)$, $J_B^-(p, q)$ ⟩/ p, q ∈P}given by $J_B^+(p, q) = \max\{\mu_B^+(p), \mu_B^+(q)\}$ and $J_B^-(p, q) = \max\{\mu_B^-(p), \mu_B^-(q)\}$,∀p, q ∈ P.

Theorem 3.5. If μ and δ are the $B_iAFI_{km}s$ of P and Q respectively, then μ x δ is a B_iAFI_{km} of P x Q.

Proof.

For any (p, q) ∈P x Q, we have

$(\mu \times \delta)_B^+(0, 0) = \max\{\mu_B^+(0), \delta_B^+(0)\}$
$\leq \max\{\mu_B^+(p), \delta_B^+(q)\}$
$= (\mu \times \delta)_B^+(p, q)$

And

$(\mu \times \delta)_B^-(0, 0) = \min\{\mu_B^-(0), \delta_B^-(0)\}$
$\geq \min\{\mu_B^-(p), \delta_B^-(q)\}$
$= (\mu \times \delta)_B^-(p, q)$

Also, let $(p_1, p_2), (q_1, q_2) \in$ P x Q.

$(\mu \times \delta)_B^+((q_1, q_2))$
$= (\mu \times \delta)_B^+(q_1, q_2)$
$= \max\{\mu_B^+(q_1), \delta_B^+(q_2)\}$
$\leq \max\{\max\{\mu_B^+(q_1 \odot p_1), \mu_B^+(p_1 \odot (p_1 \odot q_1)\}\{\delta_B^+(q_2 \odot p_2), \delta_B^+(p_2 \odot (p_2 \odot q_2)\}$
$\leq \max\{\max\{\mu_B^+(q_1 \odot p_1), \delta_B^+(q_2 \odot p_2),)\}, \max\{\mu_B^+(p_1 \odot (p_1 \odot q_1)), \delta_B^+(p_2 \odot (p_2 \odot q_2)\}\}$ $=$ $\max\{(\mu \times \delta)_B^+(q_1 \odot p_1), (q_2 \odot p_2)(\mu \times \delta)_B^+((p_1 \odot (p_1 \odot q_1), (p_2 \odot (p_2 \odot q_2))\}\}$
And
$(\mu \times \delta)_B^-((q_1, q_2))$
$= (\mu \times \delta)_B^-(q_1, q_2)$
$= \min\{\mu_B^-(q_1), \delta_B^-(q_2)\}$
$\geq \min\{\min\{\mu_B^-(q_1 \odot p_1), \mu_B^-(p_1 \odot (p_1 \odot q_1)\}\{\delta_B^-(q_2 \odot p_2), \delta_B^-(p_2 \odot (p_2 \odot q_2)\}$
$\geq \min\{\min\{\mu_B^-(q_1 \odot p_1), \delta_B^-(q_2 \odot p_2),)\}, \min\{\mu_B^-(p_1 \odot (p_1 \odot q_1)), \delta_B^-(p_2 \odot (p_2 \odot q_2)\}\}$
$= \min\{(\mu \times \delta)_B^-(q_1 \odot p_1), (q_2 \odot p_2)(\mu \times \delta)_B^-((p_1 \odot (p_1 \odot q_1), (p_2 \odot (p_2 \odot q_2))\}\}$
Therefore μ x δ is a B_iAFI_{km} of P x Q.

Theorem 3.6. For any given B_iAFS of a K-algebra P, let J be the strongest bipolar valued fuzzy relation on P. If μ is a B_iAFI_{km}ofPxP, then $J_B^+(P, P) \geq J_B^+(0, 0), \forall p \in P$ and
$J_B^-(P, P) \leq J_B^-(0, 0), \forall p \in P$.
Proof.
Here J is the strongest bipolar valued fuzzy relation on P x P, then
$J_B^+(P, P) = \max\{\mu_B^+(p), \mu_B^+(p)\}$
$\geq \max\{\mu_B^+(0), \mu_B^+(0)\}$
$= J_B^+(0, 0), \forall p \in P$.
$\Rightarrow J_B^+(P, P) \geq J_B^+(0, 0), \forall p \in P$
In the same way,
$J_B^-(P, P) = \min\{\mu_B^-(p), \mu_B^-(p)\}$
$\leq \min\{\mu_B^-(0), \mu_B^-(0)\}$
$= J_B^-(0, 0), \forall p \in P$.
$\Rightarrow J_B^-(P, P) \leq J_B^-(0, 0), \forall p \in P$.

Theorem 3.7. Let μ be the B_iAFS in a K-Algebra P and J be the strongest bipolar valued fuzzy relation on P. If μ is a B_iAFI_{km} of P iff J is a B_iAFI_{km} of P x P.

Proof.
Suppose μ is a B_iAFI_{km} of P.
Then $J_B^+(0, 0) = \max\{\mu_B^+(0), \mu_B^+(0)\}$
$\leq \max\{\mu_B^+(p), \mu_B^+(q)\}$
$= J_B^+(p, q), \forall p, q \in P$.
And $J_B^-(0, 0) = \min\{\mu_B^-(0), \mu_B^-(0)\}$
$\geq \min\{\mu_B^-(p), \mu_B^-(q)\}$
$= J_B^-(p, q), \forall p, q \in P$.
Also for any $(p_1, p_2), (q_1, q_2) \in P \times P$

$J_B^+(q_1, q_2) = \max\{\mu_B^+(q_1), \mu_B^+(q_2)\}$
$\leq \max\{\{\max\{\mu_B^+(q_1 \odot p_1), \mu_B^+(p_1 \odot (p_1 \odot q_1)\}\{\delta_B^+(q_2 \odot p_2), \delta_B^+(p_2 \odot (p_2 \odot q_2)\}$
$\leq \max\{\max\{\mu_B^+(q_1 \odot p_1), \delta_B^+(q_2 \odot p_2),)\}, \max\{\mu_B^+(p_1 \odot (p_1 \odot q_1)), \delta_B^+(p_2 \odot (p_2 \odot q_2)\}\}$
$= \max\{(J_B^+(q_1 \odot p_1), (q_2 \odot p_2))\}\{J_B^+((p_1 \odot (p_1 \odot q_1), (p_2 \odot (p_2 \odot q_2))\}$

And also

$J_B^-(q_1, q_2) = \min\{ \mu_B^-(q_1), \mu_B^-(q_2) \}$

$\geq \min \{\{\min\{ \mu_B^-(q_1 \odot p_1), \mu_B^- (p_1 \odot (p_1 \odot q_1)\}\{ \delta_B^- (q_2 \odot p_2), \delta_B^-(p_2 \odot (p_2 \odot q_2)\}$

$\geq \min \{\min\{ \mu_B^- (q_1 \odot p_1), \delta_B^- (q_2 \odot p_2),)\},\min\{ \mu_B^- (p_1 \odot (p_1 \odot q_1)),\delta_B^- (p_2 \odot (p_2 \odot q_2)\}\}$

$= \min\{(J_B^-(q_1 \odot p_1), (q_2 \odot p_2))\}\{J_B^-((p_1 \odot (p_1 \odot q_1),(p_2 \odot (p_2 \odot q_2))\}$

Therefore,J is a B_iAFI_{km} of P x P.

Conversely, suppose that J is a B_iAFI_{km} of P x P, then

$J_B^+(P, P) \geq J_B^+(0, 0)$ where (0,0) is the zero element of P x P.

Which means that

$\max \{ \mu_B^+(p) , \mu_B^+(q) \} \geq \max \{ \mu_B^+(0) , \mu_B^+(0) \}$

$\Rightarrow \mu_B^+(p) \geq \mu_B^+(0), \forall\ p \in P$ and also $\mu_B^-(p) \leq \mu_B^-(0), \forall\ p \in P$.

Now, let $(p_1, p_2), (q_1, q_2) \in P \times P$

Then,

$\max \{ \mu_B^+(q_1),\mu_B^+(q_2) \} = J_B^+(q_1, q_2)$

$\leq \max\{\{(J_B^+(q_1 \odot p_1), (q_2 \odot p_2))\}\{J_B^+((p_1 \odot (p_1 \odot q_1),(p_2 \odot (p_2 \odot q_2))\}$

$= \max \{\max\{ \mu_B^+ (q_1 \odot p_1), \delta_B^+ (q_2 \odot p_2),)\},\max\{ \mu_B^+ (p_1 \odot (p_1 \odot q_1)),\delta_B^+ (p_2 \odot (p_2 \odot q_2)\}\}$

In particular, if we take $p_2=q_2=0$,then

$\mu_B^+(q_1) \leq \max \{ \mu_B^+(q_1),\mu_B^+(q_2) \}$

Also, $\min \{ \mu_B^-(q_1),\mu_B^-(q_2) \} = J_B^-(q_1, q_2)$

$\geq \min\{\{(J_B^-(q_1 \odot p_1), (q_2 \odot p_2))\}\{J_B^-((p_1 \odot (p_1 \odot q_1),(p_2 \odot (p_2 \odot q_2))\}$

$= \min \{\min\{ \mu_B^- (q_1 \odot p_1), \delta_B^- (q_2 \odot p_2),)\},\min\{ \mu_B^- (p_1 \odot (p_1 \odot q_1)),\delta_B^- (p_2 \odot (p_2 \odot q_2)\}\}$

In particular, if we take $p_2=q_2=0$,then

$\mu_B^-(q_1) \geq \min \{ \mu_B^-(q_1),\mu_B^-(q_2) \}$

This proves μ is a B_iAFI_{km} of P.

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