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Approximate Solutions Of Transient Free - Convective Flow In A Vertical Channel Due To Symmetric Heating Using Laplace Transform Technique

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Abstract
This paper presents a closed form solution for transient free convective flow of a viscous and incompressible fluid in a vertical channel due to symmetric heating of channel walls. Laplace transform technique has been used to obtain the expression for velocity and temperature fields by solving the governing differential equations. The influence of physical parameters on the velocity field, skin-friction, rate of heat transfer and volumetric flux of fluid are carefully analysed. Correlation between steady state time and Prandtl number has been developed. It is observed that nature of correlation is linear when Prandtl number is greater than one while cubic for Prandtl number less than one. Also the analytical expressions here derived are in satisfactory.
Key words: free convection, symmetric heating, Laplace transform, Prandtl number.

Introduction

Steady laminar free-convective flow through a vertical channel has been studied extensively by many authors [cf. Ostrach (1952); Sparrow et al. (1959)]. Also Ostrach (1954) studied the combined effects of steady free and forced convective laminar flow and heat transfer in a vertical channel. Aung et al. (1972) and Aung (1972), Miyatake and Fuzii (1972, 1973) presented their results for steady free convection flow in vertical channel for different physical situations.
Transient behaviour of natural convection flow between infinite vertical parallel surface has its application in technological processes such as the early stages of melting and in transient heating of insulating air gaps by heat input at the start-up of furnaces. Also, unsteady laminar free convection flow is likely to find wider use as it could provide the flow mechanism in some type of solar heating and ventilating passive systems. To know the flow behaviour of transient free convective flow, Singh (1988) and Singh et al. (1996) studied the free convective flow in a vertical channel when one of the channel wall is moving impulsively and asymmetric heating of the plate respectively. Paul et al. (1996) studied the transient free convective flow in a vertical channel with constant temperature and constant heat flux on the walls. So this paper is devoted to analyse the transient behaviour of natural convection in vertical channel due to symmetric heating of the walls of channel.

Visuvasam et al. (2018), a nonlinear diffusion equations are performed to study the chronoamperometric limiting current generated from the electrochemical reaction in a rotating disk electrode for second-order ECE reactions when the chemical step is irreversible.
Thamizh Suganya et al. (2020) discussed about a mathematical model of the magnetohydrodynamic free convective flow of a viscous incompressible fluid, which is based on a system of coupled steady-state nonlinear deferential equations and also analysis the sensitivity. Visuvasam et al. (2018) determined the current generated from the electrochemical reaction in a porous rotating disk electrode (PRDE) is derived when the reactant transport is dominated by advection and diffusion with the limiting case of low rotation rates. Preethi et al. (2017), non stationary diffusion equation containing a non linear term related to Michaelis-Menten and ping-pong kinetics.

2. Mathematical Analysis

An unsteady free-convective flow through a vertical channel is considered. The x′-axis is taken along the vertical direction and y′-axis perpendicular to it. At time $t' \le 0$, the fluid and channel walls are at same temperature. At time $t' > 0$, the temperature of the channel walls is instantaneously raised or lowered to T_1 which is thereafter maintained constant. Also it is assumed that all the fluid properties are constant except that the influence of the density variation with temperature is considered in the body force term. Then the flow is governed by the following system of non-dimensional equations:

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + T\,, \tag{1}$$

$$\frac{\partial T}{\partial t} = \frac{1}{\Pr}\frac{\partial^2 T}{\partial y^2}\,. \tag{2}$$

The non-dimensional quantities used in the above equations are:

$$t = \frac{t'\nu}{h^2}, \qquad y = \frac{y'}{h}, \qquad u = \frac{u'\nu}{g\beta(T_1 - T_0)h^2},$$

$$T = \frac{T' - T_0}{T_1 - T_0}, \qquad \Pr = \frac{\mu Cp}{k}\,. \tag{3}$$

The physical quantities used in the above equations are defined in nomenclature. As the flow is symmetrical about the plane y = 0, the initial and boundary conditions assume the following form

$$t \le 0: \quad u(y,t) = 0; \qquad T(y,t) = 0 \qquad \text{for } 0 \le y \le 1,$$

$$t > 0: \quad \left|\begin{array}{ll} u(1,t) = 0; & T(1,t) = 1, \\[2ex] \dfrac{\partial u}{\partial y}(0,t) = 0; & \dfrac{\partial T}{\partial y}(0,t) = 0. \end{array}\right. \tag{4}$$

The solution of Eqs. (1) and (2) under the conditions (4) using the Laplace transform technique is obtained as

$$u = \frac{1}{(\Pr - 1)}\sum_{n=0}^{\infty}(-1)^n\left[t\left\{F_1(a,\ 1.0) + F_1(b,\ 1.0) - F_1(a,\ \Pr) - F_1(b,\ \Pr)\right\}\right], \tag{5}$$

$$T = \sum_{n=0}^{\infty}(-1)^n\left[erfc\left(b\sqrt{\Pr}\right) + erfc\left(a\sqrt{\Pr}\right)\right], \tag{6}$$

where,

$$a_1 = 2n + 1 - y, \qquad b_1 = 2n + 1 + y, \qquad a = \frac{a_1}{2\sqrt{t}}, \qquad b = \frac{b_1}{2\sqrt{t}}.$$

The functional F_1 used in Eq. (5) is defined as follows:

$$F_1(d_1; d_2) = \left[\left(1 + 2d_1^2 d_2\right)\, erfc\left(d_1\sqrt{d_2}\right) - \frac{2d_1\sqrt{d_2}}{\sqrt{\pi}}\exp\left(-d_1^2 d_2\right)\right].$$

Using the expressions (5) and (6) the skin-friction and the rate of heat transfer at the wall y = 1 in non-dimensional form are given by

$$\tau = \left.\frac{\partial u}{\partial y}\right|_{y=1} = \frac{1}{(\Pr - 1)}\sum_{n=0}^{\infty}(-1)^n\left[F_2(n;\ 1.0;\ t) - F_2(n;\ \Pr;\ t) - F_2(n+1;\ 1.0;\ t)\right.$$
$$\left. + F_2(n+1;\ \Pr;\ t)\right], \tag{7}$$

where

$$F_2(d_1,\ d_2,\ d_3) = 2\sqrt{\frac{d_3 d_2}{\pi}}\ \exp\left(-\frac{d_1^2 d_3}{d_3}\right) - 2\ d_1\ d_2\ erfc\left(d_1\sqrt{\frac{d_2}{d_3}}\right),$$

and

$$Nu = -\frac{\partial T}{\partial y}\bigg|_{y=1} = \sum_{n=0}^{\infty}(-1)^{n+1}\sqrt{\frac{\Pr}{\pi t}}\left[\exp\left\{\frac{-n^2}{t}\right\} - \exp\left\{\frac{(n+1)^2}{t}\right\}\right]. \qquad (8)$$

Using the expression,

$$Q = \int_0^1 u dy \qquad (9)$$

volumetric flux of the fluid is obtained numerically.

3. Results and Discussion

In order to analyse the transient behaviour of free-convection on transport phenomena, numerical computations are carried out for Eqs. (5) - (9) from which the velocity and temperature fields are shown in Figs. 1 and 2, respectively whereas the numerical values of volumetric flux of fluid, skin-friction and rate of heat transfer are presented in tabular form. In the calculations, the values of Prandtl number (Pr) are taken as 0.71 and 7.0 corresponding to realistic fluids air and water, respectively. From Fig. 1, it is clear that the velocity increases with increase of time parameter (t) while decreases with increase of Pr. The graph of temperature profiles (Fig. 2) reflects that the temperature increases with increase of time parameter (t) while decreases with increase of Pr. The time taken by the velocity and temperature fields to reach their steady state values are more in case of water than air. A close study of the table indicates that the skin-friction increases with time while decreases with the Prandtl number. The rate of heat transfer expressed as Nusselt number Nu, increases with increase of Pr. So we conclude that the rate of heat transfer is more in case of water than air. Also from this table we observe that the volume flux for air and water increase with time.

Further numerical calculations are carried out for different values of Pr in order to establish a correlation between steady state time (t^*) and Pr. The steady state time is obtained until the following convergence criterion is satisfied:

$$\frac{\sum_{i=1}^{11}\left|\psi_t(i) - \psi_{st}(i)\right|}{\sum_{i=1}^{11}\left|\psi_{st}(i)\right|} \le 10^{-4},$$

where ψ stands for *u* or *T* while the subscripts *t* and *st* denote the transient and steady state solution, respectively. An overall view of steady state time (t^*) has indicated that correlation between steady state time and Prandtl number can be achieved by considering it for two different cases (Pr < 1 and Pr > 1). For these cases, they are as follows:

Case 1: When Pr < 1

The overall data for steady state times gives the following cubic relation

$t^* \approx 0.905\ \Pr^3 - 0.58\ \Pr^2 + 0.711\ \Pr + 3.72$

Case 2: When Pr > 1

In this case, the steady state time is governed by the linear relation

$t^* \approx 3.69\ \Pr + 2.08$

Figures 3a and 3b show the steady state time for the cases Pr < 1 and Pr > 1 respectively. The broken curves are corresponding to the obtained values whereas the continuous curves are from the correlation given by Eqs (11) and (12). The good agreement between them indicates that the obtained correlation can be used to obtained the steady state time for fluid flow in vertical channels.

4. Conclusion

In this paper, Steady state time is more in case of water than air. The correlation development between steady state time and Prandtl number has dual nature which is cubic for Pr < 1 and linear for Pr >1. When the nature of correlation is linear when Prandtl number is greater than one while cubic for Prandtl number less than one is discussed. The Laplace transform method is applied to solve the system of non-dimensional differential equations in different dynamic models different cases for Prandtl number.

Nomenclature

Cp	specific heat at constant pressure
g	acceleration due to gravity
h	half width of the channel
k	thermal conductivity
Pr	Prandtl number
t'	time
t	dimensionless time
T_0	initial temperature of the fluid
T'	temperature of the fluid
T	temperature of the fluid in non-dimensional form
T_1	temperature of the channel walls ($t > 0$)
u'	velocity of the fluid
u	dimensionless fluid velocity
y'	co-ordinate perpendicular to the plate
y	dimensionless co-ordinate perpendicular to the plate
Greek symbols	
β	coefficient of thermal expansion
μ	dynamic viscosity
ν	kinematics viscosity

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