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DIRECT PRODUCT OF INTUITIONISTIC FUZZY HX RING

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ABSTRACT : In this paper, we define the notion of an intuitionistic fuzzy HX subring of a HX ring, Cartesian product of an intuitionistic fuzzy HX subring and some of their related properties are investigated. We define the necessity and possibility operators of an intuitionistic fuzzy subset of an intuitionistic fuzzy HX ring and discuss some of its properties.
***Keywords*:** intuitionistic fuzzy set, fuzzy HX ring, intuitionistic fuzzy HX ring, Product of intuitionistic fuzzy HX ring.

INTRODUCTION

In 1965, Zadeh [6] introduced the concept of fuzzy subset μ of a set X as a function from X into the closed unit interval [0, 1] and studied their properties. In 1988, Professor Li Hong Xing proposed the concept of HX ring and derived some of its properties, then Professor Zhong [2,3] gave the structures of HX ring on a class of ring. R.Muthuraj et.al[5]., introduced the concept of fuzzy HX ring. In this paper we define a new algebraic structure of an intuitionistic fuzzy sub HX ring of a HX ring and investigate some related properties. We define the necessity and possibility operators of an intuitionistic fuzzy subset of an intuitionistic fuzzy HX ring and discuss some of its properties. Also we introduce the image and pre-image of an intuitionistic fuzzy set in an intuitionistic fuzzy HX ring and discuss some of its properties.

PRELIMINARIES

In this section, we site the fundamental definitions that will be used in the sequel. Throughout this paper, R = (R ,+, ·) is a Ring, e is the additive identity element of R and xy, we mean x.y

2.1 Definition

Let R be a ring. In 2^R - {ϕ}, a non-empty set $\vartheta \subset 2^R$ - {ϕ} with two binary operation ' + ' and '.' is said to be a HX ring on R if ϑ is a ring with respect to the algebraic operation defined by

i. $A + B = \{a + b / a \in A \text{ and } b \in B\}$, which its null element is denoted by Q , and the negative element of A is denoted by – A.
ii. $AB = \{ab / a \in A \text{ and } b \in B\}$,
iii. A (B + C) = AB + AC and (B + C) A = BA + CA.

3. Properties of an intuitionistic fuzzy HX ring

3.1 Definition

Let R be a ring. Consider H = { x, $\mu(x)$, $\nu(x)$/ $x\in R$ } be an intuitionistic fuzzy set defined on a ring R, where $\mu : R\to[0,1]$, $\nu : R\to[0,1]$ such that $0 \le \mu(x) + \nu(x) \le 1$. Let $\vartheta \subset 2^{R} - \{\phi\}$ be a HX ring. An intuitionistic fuzzy subset $\lambda^{H} = \langle A , \lambda^{\mu}(A) , \lambda^{\nu}(A) / A\in\vartheta, 0 \le \lambda^{\mu}(A) + \lambda^{\nu}(A) \le 1 \rangle$ of a HX ring ϑ is said to be an intuitionistic fuzzy HX (IFHX) ring if for all $A,B\in\vartheta$

(i). $\lambda^{\mu}(A-B) \ge \min\{ \lambda^{\mu}(A) , \lambda^{\mu}(B)\}$ (ii). $\lambda^{\mu}(AB) \ge \min\{ \lambda^{\mu}(A) , \lambda^{\mu}(B)\}$,

(iii). $\lambda^{\gamma}(A-B) \le \max\{ \lambda^{\gamma}(A) , \lambda^{\gamma}(B)\}$ (iv). $\lambda^{\gamma}(AB) \le \max\{ \lambda^{\gamma}(A) , \lambda^{\gamma}(B)\}$

Where $\lambda^{\mu}(A) = \max\{ \mu(x) / x\in A\subseteq R\}$, $\lambda^{\gamma}(A) = \min\{\gamma(x) / x\in A\subseteq R\}$.

3.2 Definition

$\psi_1 = \{\langle A , \lambda^{\mu}(A) , \lambda^{\gamma}(A) \rangle / A\in\vartheta \}$ and $\psi_2 = \{\langle A , \Theta^{\mu}(A) , \Theta^{\gamma}(A) \rangle / A\in\vartheta \}$ be two intuitionistic fuzzy HX rings of a HX ring ϑ,then the product $\psi_1 \times \psi_2$ is defined as

$$(\psi_1\times \psi_2) = \{\langle(A,B), (\lambda^{\mu} \times \Theta^{\mu}) (A,B), (\lambda^{\gamma} \times \Theta^{\gamma}) (A, B)\rangle /(A, B) \in \mathfrak{R}_1 \times \mathfrak{R}_2\},$$

where, $(\lambda^{\mu} \times \Theta^{\mu})(A, B) = \min \{ \lambda^{\mu} (A), \Theta^{\mu} (B)\}$, for all $(A, B) \in \mathfrak{R}_1 \times \mathfrak{R}_2$,

$(\lambda^{\gamma} \times \Theta^{\gamma})(A, B) = \max \{ \lambda^{\gamma} (A), \Theta^{\gamma} (B)\}$, for all $(A, B) \in \mathfrak{R}_1 \times \mathfrak{R}_2$.

3.3 Theorem

If ψ_1 and ψ_2 be two intuitionistic fuzzy HX rings of a HX ring ϑ, then $\psi_1 \times \psi_2$ is also intuitionistic fuzzy HX ring of a HX ring $\vartheta\times\vartheta$.

Proof Let $\psi_1 = \{\langle A , \lambda^{\mu}(A) , \lambda^{\gamma}(A) \rangle / A\in\vartheta \}$ and $\psi_2 = \{\langle A , \Theta^{\mu}(A) , \Theta^{\gamma}(A) \rangle / A\in\vartheta \}$ be two intuitionistic fuzzy HX rings of a HX ring ϑ.

To Prove that $\psi_1 \times \psi_2$ is also an intuitionistic fuzzy HX ring of a HX ring $\vartheta\times\vartheta$.

For any A,B ,C & $D\in\vartheta$,we have

(i) $(\lambda^{\mu} \times \Theta^{\mu}) ((A,B) - (C,D)) = (\lambda^{\mu} \times \Theta^{\mu}) ((A-C), (B-D))\}$

$= \min\{ \lambda^{\mu}(A-C), \Theta^{\mu} (B-D)\}$

$\ge \min\{ \min \{ \lambda^{\mu} (A) , \lambda^{\mu} (C)\}, \min\{ \Theta^{\mu} (B) , \Theta^{\mu} (D)\}\}$

$\ge \min\{ \min \{ \lambda^{\mu} (A) , \Theta^{\mu} (B)\}, \min\{ \lambda^{\mu} (C), \Theta^{\mu} (D)\}\}$

$= \min \{ (\lambda^{\mu} \times \Theta^{\mu}) (A,B) , (\lambda^{\mu} \times \Theta^{\mu}) (C,D)\}$

Hence, $(\lambda^{\mu} \times \Theta^{\mu}) ((A,B) - (C,D)) \ge \min \{ (\lambda^{\mu} \times \Theta^{\mu}) (A,B) , (\lambda^{\mu} \times \Theta^{\mu}) (C,D)\}$

(ii) $(\lambda^{\mu} \times \Theta^{\mu}) ((A,B) . (C,D)) = (\lambda^{\mu} \times \Theta^{\mu}) ((AC, BD))$

$= \min\{ \lambda^{\mu}(AC), \Theta^{\mu} (BD)\}$

$\ge \min\{ \min \{ \lambda^{\mu} (A) , \lambda^{\mu} (C)\}, \min\{ \Theta^{\mu} (B) , \Theta^{\mu} (D)\}\}$

$\ge \min\{ \min \{ \lambda^{\mu} (A) , \Theta^{\mu} (B)\},\min\{ \lambda^{\mu} (C), \Theta^{\mu} (D)\}$

$= \min \{ (\lambda^{\mu} \times \Theta^{\mu}) (A,B) , (\lambda^{\mu} \times \Theta^{\mu}) (C,D)\}$

Hence, $(\lambda^{\mu} \times \Theta^{\mu}) ((A,B) . (C,D)) \ge \min \{ (\lambda^{\mu} \times \Theta^{\mu}) (A,B) , (\lambda^{\mu} \times \Theta^{\mu}) (C,D)\}$

(iii) $(\lambda^{\gamma} \times \Theta^{\gamma})((A,B) - (C,D)) = (\lambda^{\gamma} \times \Theta^{\gamma})((A-C), (B-D))$

$\le \max\{ \lambda^{\gamma} (A-C), \Theta^{\gamma} (B-D)\}$

$= \max\{ \max \{ \lambda^{\gamma} (A) , \lambda^{\gamma} (C)\}, \max\{ \Theta^{\gamma} (B) , \Theta^{\gamma} (D)\}\}$

$= \max\{ \max \{ \lambda^{\gamma} (A) , \Theta^{\gamma} (B)\},\max\{ \lambda^{\gamma} (C) , \Theta^{\gamma} (D)\}\}$

Hence, $(\lambda^{\gamma} \times \Theta^{\gamma})((A,B) - (C,D)) \le \max \{ (\lambda^{\gamma} \times \Theta^{\gamma}) (A,B) , (\lambda^{\gamma} \times \Theta^{\gamma}) (C,D)\}$

(iv) $(\lambda^{\gamma} \times \Theta^{\gamma})((A,B).(C,D)) = (\lambda^{\gamma} \times \Theta^{\gamma})((AC, BD))$
$\leq \max\{\lambda^{\gamma}(AC), \Theta^{\gamma}(BD)\}$
$= \max\{\max\{\lambda^{\gamma}(A), \lambda^{\gamma}(C)\}, \max\{\Theta^{\gamma}(B), \Theta^{\gamma}(D)\}\}$
$= \max\{\max\{\lambda^{\gamma}(A), \Theta^{\gamma}(B)\}, \max\{\lambda^{\gamma}(C), \Theta^{\gamma}(D)\}\}$
$= \max\{(\lambda^{\gamma} \times \Theta^{\gamma})(A,B), (\lambda^{\gamma} \times \Theta^{\gamma})(C,D)\}$

Hence , $(\lambda^{\gamma} \times \Theta^{\gamma})((A,B).(C,D)) \leq \max\{(\lambda^{\gamma} \times \Theta^{\gamma})(A,B), (\lambda^{\gamma} \times \Theta^{\gamma})(C,D)\}$

Therefore , $\psi_1 \times \psi_2$ is an intuitionistic fuzzy HX rings of a HX ring $\vartheta \times \vartheta$.

3.4 Definition

Let $\psi = \{\langle A, \lambda^{\mu}(A), \lambda^{\gamma}(A) \rangle / \text{ for all } A \in \vartheta\}$ be an intuitionistic fuzzy subset of a HX ring ϑ . We define the following "necessity" and "possibility" operations :

$\Box\, \psi = \{\langle A, \lambda^{\mu}(A), 1-\lambda^{\mu}(A) \rangle / A \in \vartheta\}$.

$\Diamond\, \psi = \{\langle A, 1-\lambda^{\gamma}(A), \lambda^{\gamma}(A) \rangle / A \in \vartheta\}$

3.5 Theorem

If $\psi_1 \times \psi_2 = \{A \times A, \lambda^{\mu} \times \Theta^{\mu}, \lambda^{\gamma} \times \Theta^{\gamma}\}$ is an intuitionistic fuzzy HX subring $\vartheta \times \vartheta$ then $\Box(\psi_1 \times \psi_2) = \{(\lambda^{\mu} \times \Theta^{\mu}), (\lambda^{\mu} \times \Theta^{\mu})^{c}\}$ is an intuitionistic fuzzy HX subring of $\vartheta \times \vartheta$.

Proof: Let $\Box(\psi_1 \times \psi_2) = \{\langle A \times A, (\lambda^{\mu} \times \Theta^{\mu})(A \times A), (\lambda^{\mu} \times \Theta^{\mu})^{c}(A \times A), / A \in \vartheta\}$

To prove that $\Box(\psi_1 \times \psi_2)$ is an intuitionistic fuzzy HX subring of $\vartheta \times \vartheta$.

Given $\psi_1 \times \psi_2$ is an intuitionistic fuzzy HX subring of $\vartheta \times \vartheta$, then

(i) $(\lambda^{\mu} \times \Theta^{\mu})((A,B)-(C,D)) \geq \min\{(\lambda^{\mu} \times \Theta^{\mu})(A,B), (\lambda^{\mu} \times \Theta^{\mu})(C,D)\}$

(ii) $(\lambda^{\mu} \times \Theta^{\mu})((A,B).(C,D)) \geq \min\{(\lambda^{\mu} \times \Theta^{\mu})(A,B), (\lambda^{\mu} \times \Theta^{\mu})(C,D)\}$

(iii) $(\lambda^{\gamma} \times \Theta^{\gamma})((A,B)-(C,D)) \leq \max\{(\lambda^{\gamma} \times \Theta^{\gamma})(A,B), (\lambda^{\gamma} \times \Theta^{\gamma})(C,D)\}$

(iv) $(\lambda^{\gamma} \times \Theta^{\gamma})((A,B).(C,D)) \leq \max\{(\lambda^{\gamma} \times \Theta^{\gamma})(A,B), (\lambda^{\gamma} \times \Theta^{\gamma})(C,D)\}$

ie , to show that

(a) $(\lambda^{\mu} \times \Theta^{\mu})^{c}((A,B)-(C,D)) \leq \max\{(\lambda^{\mu} \times \Theta^{\mu})^{c}(A,B), (\lambda^{\mu} \times \Theta^{\mu})^{c}(C,D)\}$

(b) $(\lambda^{\mu} \times \Theta^{\mu})^{c}((A,B).(C,D)) \leq \max\{(\lambda^{\mu} \times \Theta^{\mu})^{c}(A,B), (\lambda^{\mu} \times \Theta^{\mu})^{c}(C,D)\}$.

Now ,

(a) $(\lambda^{\mu} \times \Theta^{\mu})^{c}((A,B)-(C,D)) = 1-(\lambda^{\mu} \times \Theta^{\mu})((A,B)-(C,D))$
$= 1-(\lambda^{\mu} \times \Theta^{\mu})\{((A-C), (B-D))\}$
$\leq 1-\min\{\lambda^{\mu}(A-C), \Theta^{\mu}(B-D)\}$
$= 1-\min\{\min\{\lambda^{\mu}(A), \lambda^{\mu}(C)\}, \min\{\Theta^{\mu}(B), \Theta^{\mu}(D)\}\}$
$= 1-\min\{\min\{\lambda^{\mu}(A), \Theta^{\mu}(B)\}, \min\{\lambda^{\mu}(C), \Theta^{\mu}(D)\}\}$
$= 1-\min\{(\lambda^{\mu} \times \Theta^{\mu})(A,B), (\lambda^{\mu} \times \Theta^{\mu})(C,D)\}$
$= \max\{1-(\lambda^{\mu} \times \Theta^{\mu})(A,B), 1-(\lambda^{\mu} \times \Theta^{\mu})(C,D)\}$
$= \max\{(\lambda^{\mu} \times \Theta^{\mu})^{c}(A,B), (\lambda^{\mu} \times \Theta^{\mu})^{c}(C,D)\}$

Hence , $(\lambda^{\mu} \times \Theta^{\mu})^{c}((A,B)-(C,D)) \leq \max\{(\lambda^{\mu} \times \Theta^{\mu})^{c}(A,B), (\lambda^{\mu} \times \Theta^{\mu})^{c}(C,D)\}$

(b) $(\lambda^{\mu} \times \Theta^{\mu})^{c}((A,B).(C,D)) = 1-(\lambda^{\mu} \times \Theta^{\mu})((A,B).(C,D))$
$= 1-(\lambda^{\mu} \times \Theta^{\mu})\{(AC, BD)\}$
$\leq 1-\min\{\lambda^{\mu}(AC), \Theta^{\mu}(BD)\}$
$= 1-\min\{\min\{\lambda^{\mu}(A), \lambda^{\mu}(C)\}, \min\{\Theta^{\mu}(B), \Theta^{\mu}(D)\}\}$
$= 1-\min\{\min\{\lambda^{\mu}(A), \Theta^{\mu}(B)\}, \min\{\lambda^{\mu}(C), \Theta^{\mu}(D)\}\}$
$= 1-\min\{(\lambda^{\mu} \times \Theta^{\mu})(A,B), (\lambda^{\mu} \times \Theta^{\mu})(C,D)\}$
$= \max\{1-(\lambda^{\mu} \times \Theta^{\mu})(A,B), 1-(\lambda^{\mu} \times \Theta^{\mu})(C,D)\}$
$= \max\{(\lambda^{\mu} \times \Theta^{\mu})^{c}(A,B), (\lambda^{\mu} \times \Theta^{\mu})^{c}(C,D)\}$

Hence , $(\lambda^\mu \times \Theta^\mu)^c((A,B).(C,D)) \le \max\{(\lambda^\mu \times \Theta^\mu)^c(A,B), (\lambda^\mu \times \Theta^\mu)^c(C,D)\}$
Therefore $\square(\psi_1 \times \psi_2) = \{\langle A\times A, (\lambda^\mu \times \Theta^\mu)(A\times A), (\lambda^\mu \times \Theta^\mu)^c(A\times A), / A\in\vartheta\}$ is an intuitionistic fuzzy HX subring of $\vartheta\times\vartheta$.

3.6 Theorem

If $\psi_1 \times \psi_2 = \{A\times A, \lambda^\mu \times \Theta^\mu, \lambda^\gamma \times \Theta^\gamma\}$ is an intuitionistic fuzzy HX ring $\vartheta\times\vartheta$ then $\Diamond(\psi_1 \times \psi_2) = \{(\lambda^\gamma \times \Theta^\gamma)^c, (\lambda^\gamma \times \Theta^\gamma)\}$ is an intuitionistic fuzzy HX ring of $\vartheta\times\vartheta$.

Proof: Let $\Diamond(\psi_1 \times \psi_2) = \{\langle A\times A, (\lambda^\gamma \times \Theta^\gamma)^c(A\times A), (\lambda^\gamma \times \Theta^\gamma)(A\times A), / A\in\vartheta\}$
To prove that $\Diamond(\psi_1 \times \psi_2)$ is an intuitionistic fuzzy HX ring of $\vartheta\times\vartheta$.
Given $\psi_1 \times \psi_2$ is an intuitionistic fuzzy HX ring of $\vartheta\times\vartheta$, then
ie , to show that
(i) $(\lambda^\gamma \times \Theta^\gamma)^c((A,B)-(C,D)) \ge \min\{(\lambda^\gamma \times \Theta^\gamma)^c(A,B), (\lambda^\gamma \times \Theta^\gamma)^c(C,D)\}$
(ii) $(\lambda^\gamma \times \Theta^\gamma)^c((A,B).(C,D)) \ge \min\{(\lambda^\gamma \times \Theta^\gamma)^c(A,B), (\lambda^\gamma \times \Theta^\gamma)^c(C,D)\}$.

Now ,
(i) $(\lambda^\gamma \times \Theta^\gamma)^c((A,B)-(C,D)) = 1-(\lambda^\gamma \times \Theta^\gamma)((A,B)-(C,D))$
$= 1-(\lambda^\gamma \times \Theta^\gamma)\{((A-C), (B-D))\}$
$\ge 1-\max\{\lambda^\gamma(A-C), \Theta^\gamma(B-D)\}$
$= 1-\max\{\max\{\lambda^\gamma(A), \lambda^\gamma(C)\}, \max\{\Theta^\gamma(B), \Theta^\gamma(D)\}\}$
$= 1-\max\{\max\{\lambda^\gamma(A), \Theta^\gamma(B)\}, \max\{\lambda^\gamma(C), \Theta^\gamma(D)\}\}$
$= 1-\max\{(\lambda^\gamma \times \Theta^\gamma)(A,B), (\lambda^\gamma \times \Theta^\gamma)(C,D)\}$
$= \min\{1-(\lambda^\gamma \times \Theta^\gamma)(A,B), 1-(\lambda^\gamma \times \Theta^\gamma)(C,D)\}$
$= \min\{(\lambda^\gamma \times \Theta^\gamma)^c(A,B), (\lambda^\gamma \times \Theta^\gamma)^c(C,D)\}$
Hence , $(\lambda^\gamma \times \Theta^\gamma)^c((A,B)-(C,D)) \ge \min\{(\lambda^\gamma \times \Theta^\gamma)^c(A,B), (\lambda^\gamma \times \Theta^\gamma)^c(C,D)\}$

(ii) $(\lambda^\gamma \times \Theta^\gamma)^c((A,B).(C,D)) = 1-(\lambda^\gamma \times \Theta^\gamma)((A,B).(C,D))$
$= 1-(\lambda^\gamma \times \Theta^\gamma)\{(AC, BD)\}$
$\ge 1-\max\{\lambda^\gamma(AC), \Theta^\gamma(BD)\}$
$= 1-\max\{\max\{\lambda^\gamma(A), \lambda^\gamma(C)\}, \max\{\Theta^\gamma(B), \Theta^\gamma(D)\}\}$
$= 1-\max\{\max\{\lambda^\gamma(A), \Theta^\gamma(B)\}, \max\{\lambda^\gamma(C), \Theta^\gamma(D)\}\}$
$= 1-\max\{(\lambda^\gamma \times \Theta^\gamma)(A,B), (\lambda^\gamma \times \Theta^\gamma)(C,D)\}$
$= \min\{1-(\lambda^\gamma \times \Theta^\gamma)(A,B), 1-(\lambda^\gamma \times \Theta^\gamma)(C,D)\}$
$= \min\{(\lambda^\gamma \times \Theta^\gamma)^c(A,B), (\lambda^\gamma \times \Theta^\gamma)^c(C,D)\}$
Hence, $(\lambda^\gamma \times \Theta^\gamma)^c((A,B).(C,D)) \ge \min\{(\lambda^\gamma \times \Theta^\gamma)^c(A,B), (\lambda^\gamma \times \Theta^\gamma)^c(C,D)\}$
Therefore, $\Diamond(\psi_1 \times \psi_2) = \{\langle A\times A, (\lambda^\gamma \times \Theta^\gamma)^c(A\times A), (\lambda^\gamma \times \Theta^\gamma)(A\times A), / A\in\vartheta\}$
is an intuitionistic fuzzy HX subring of $\vartheta\times\vartheta$.

3.7 Theorem

If $\psi_1 \times \psi_2 = \{A\times A, \lambda^\mu \times \Theta^\mu, \lambda^\gamma \times \Theta^\gamma\}$ is an intuitionistic fuzzy HX ring $\vartheta\times\vartheta$ if and only if the fuzzy subsets $\lambda^\mu \times \Theta^\mu$ and $(\lambda^\gamma \times \Theta^\gamma)^c$ are fuzzy HX ring of a HX ring $\vartheta\times\vartheta$.

Proof: Given $\psi_1 \times \psi_2$ is an intuitionistic fuzzy HX ring of $\vartheta\times\vartheta$, then
(i) $(\lambda^\mu \times \Theta^\mu)((A,B)-(C,D)) \ge \min\{(\lambda^\mu \times \Theta^\mu)(A,B), (\lambda^\mu \times \Theta^\mu)(C,D)\}$
(ii) $(\lambda^\mu \times \Theta^\mu)((A,B).(C,D)) \ge \min\{(\lambda^\mu \times \Theta^\mu)(A,B), (\lambda^\mu \times \Theta^\mu)(C,D)\}$
(iii) $(\lambda^\gamma \times \Theta^\gamma)((A,B)-(C,D)) \le \max\{(\lambda^\gamma \times \Theta^\gamma)(A,B), (\lambda^\gamma \times \Theta^\gamma)(C,D)\}$
(iv) $(\lambda^\gamma \times \Theta^\gamma)((A,B).(C,D)) \le \max\{(\lambda^\gamma \times \Theta^\gamma)(A,B), (\lambda^\gamma \times \Theta^\gamma)(C,D)\}$
Clearly, $(\lambda^\mu \times \Theta^\mu)$ is a fuzzy HX ring of $\vartheta\times\vartheta$ by (i) and (ii) .
Now we have to show $(\lambda^\gamma \times \Theta^\gamma)^c$ is a fuzzy HX ring of $\vartheta\times\vartheta$.

ie , to show that

(i) $(\lambda^\gamma \times \Theta^\gamma)^c((A,B)-(C,D)) \geq \min\{(\lambda^\gamma \times \Theta^\gamma)^c(A,B), (\lambda^\gamma \times \Theta^\gamma)^c(C,D)\}$

(ii) $(\lambda^\gamma \times \Theta^\gamma)^c((A,B).(C,D)) \geq \min\{(\lambda^\gamma \times \Theta^\gamma)^c(A,B), (\lambda^\gamma \times \Theta^\gamma)^c(C,D)\}$.

Now ,

(i) $(\lambda^\gamma \times \Theta^\gamma)^c((A,B)-(C,D)) = 1-(\lambda^\gamma \times \Theta^\gamma)((A,B)-(C,D))$
$= 1-(\lambda^\gamma \times \Theta^\gamma)\{((A-C),(B-D))\}$
$\geq 1-\max\{\lambda^\gamma(A-C), \Theta^\gamma(B-D)\}$
$= 1-\max\{\max\{\lambda^\gamma(A), \lambda^\gamma(C)\}, \max\{\Theta^\gamma(B), \Theta^\gamma(D)\}\}$
$= 1-\max\{\max\{\lambda^\gamma(A), \Theta^\gamma(B)\}, \max\{\lambda^\gamma(C), \Theta^\gamma(D)\}\}$
$= 1-\max\{(\lambda^\gamma \times \Theta^\gamma)(A,B), (\lambda^\gamma \times \Theta^\gamma)(C,D)\}$
$= \min\{1-(\lambda^\gamma \times \Theta^\gamma)(A,B), 1-(\lambda^\gamma \times \Theta^\gamma)(C,D)\}$
$= \min\{(\lambda^\gamma \times \Theta^\gamma)^c(A,B), (\lambda^\gamma \times \Theta^\gamma)^c(C,D)\}$

Hence, $(\lambda^\gamma \times \Theta^\gamma)^c((A,B)-(C,D)) \geq \min\{(\lambda^\gamma \times \Theta^\gamma)^c(A,B), (\lambda^\gamma \times \Theta^\gamma)^c(C,D)\}$

(ii).$(\lambda^\gamma \times \Theta^\gamma)^c((A,B).(C,D)) = 1-(\lambda^\gamma \times \Theta^\gamma)((A,B).(C,D))$
$= 1-(\lambda^\gamma \times \Theta^\gamma)\{(AC, BD)\}$
$\geq 1-\max\{\lambda^\gamma(AC), \Theta^\gamma(BD)\}$
$= 1-\max\{\max\{\lambda^\gamma(A), \lambda^\gamma(C)\}, \max\{\Theta^\gamma(B), \Theta^\gamma(D)\}$
$= 1-\max\{\max\{\lambda^\gamma(A), \Theta^\gamma(B)\}, \max\{\lambda^\gamma(C), \Theta^\gamma(D)\}$
$= 1-\max\{(\lambda^\gamma \times \Theta^\gamma)(A,B), (\lambda^\gamma \times \Theta^\gamma)(C,D)\}$
$= \min\{1-(\lambda^\gamma \times \Theta^\gamma)(A,B), 1-(\lambda^\gamma \times \Theta^\gamma)(C,D)\}$
$= \min\{(\lambda^\gamma \times \Theta^\gamma)^c(A,B), (\lambda^\gamma \times \Theta^\gamma)^c(C,D)\}$

Hence, $(\lambda^\gamma \times \Theta^\gamma)^c((A,B).(C,D)) \geq \min\{(\lambda^\gamma \times \Theta^\gamma)^c(A,B), (\lambda^\gamma \times \Theta^\gamma)^c(C,D)\}$

Therefore , $\lambda^\mu \times \Theta^\mu$ and $(\lambda^\gamma \times \Theta^\gamma)^c$ are fuzzy HX ring of a HX ring $\vartheta\times\vartheta$.

Conversely, Let $\lambda^\mu \times \Theta^\mu$ and $(\lambda^\gamma \times \Theta^\gamma)^c$ are fuzzy HX ring of a HX ring

To prove that $\psi_1 \times \psi_2$ is an intuitionistic fuzzy HX ring of $\vartheta\times\vartheta$

Now we know that ,

$(\lambda^\gamma \times \Theta^\gamma)^c((A,B)-(C,D)) \geq \min\{(\lambda^\gamma \times \Theta^\gamma)^c(A,B), (\lambda^\gamma \times \Theta^\gamma)^c(C,D)\}$
$1-(\lambda^\gamma \times \Theta^\gamma)((A,B)-(C,D)) \geq \min\{1-(\lambda^\gamma \times \Theta^\gamma)(A,B), 1-(\lambda^\gamma \times \Theta^\gamma)(C,D)\}$
$1-(\lambda^\gamma \times \Theta^\gamma)((A,B)-(C,D)) = 1-\max\{(\lambda^\gamma \times \Theta^\gamma)(A,B), (\lambda^\gamma \times \Theta^\gamma)(C,D)\}$
$(\lambda^\gamma \times \Theta^\gamma)((A,B)-(C,D)) \leq \max\{(\lambda^\gamma \times \Theta^\gamma)(A,B), (\lambda^\gamma \times \Theta^\gamma)(C,D)\}$

Also

$(\lambda^\gamma \times \Theta^\gamma)^c((A,B).(C,D)) \geq \min\{(\lambda^\gamma \times \Theta^\gamma)^c(A,B), (\lambda^\gamma \times \Theta^\gamma)^c(C,D)\}$
$1-(\lambda^\gamma \times \Theta^\gamma)((A,B).(C,D)) \geq \min\{1-(\lambda^\gamma \times \Theta^\gamma)(A,B), 1-(\lambda^\gamma \times \Theta^\gamma)(C,D)\}$
$1-(\lambda^\gamma \times \Theta^\gamma)((A,B).(C,D)) = 1-\max\{(\lambda^\gamma \times \Theta^\gamma)(A,B), (\lambda^\gamma \times \Theta^\gamma)(C,D)\}$
$(\lambda^\gamma \times \Theta^\gamma)((A,B).(C,D)) \leq \max\{(\lambda^\gamma \times \Theta^\gamma)(A,B), (\lambda^\gamma \times \Theta^\gamma)(C,D)\}$

Already we have

$(\lambda^\mu \times \Theta^\mu)((A,B)-(C,D)) \geq \min\{(\lambda^\mu \times \Theta^\mu)(A,B), (\lambda^\mu \times \Theta^\mu)(C,D)\}$
$(\lambda^\mu \times \Theta^\mu)((A,B).(C,D)) \geq \min\{(\lambda^\mu \times \Theta^\mu)(A,B), (\lambda^\mu \times \Theta^\mu)(C,D)\}$

Hence,$\psi_1 \times \psi_2 = \{A\times A, \lambda^\mu \times \Theta^\mu, \lambda^\gamma \times \Theta^\gamma\}$ is an intuitionistic fuzzy HX ring $\vartheta\times\vartheta$

3.8 Theorem

If $\psi_1 \times \psi_2 = \{A\times A, \lambda^{\mu} \times \Theta^{\mu}, \lambda^{\gamma} \times \Theta^{\gamma}\}$ is an intuitionistic fuzzy HX ring $\vartheta\times\vartheta$ if and only if the fuzzy subsets $(\lambda^{\mu} \times \Theta^{\mu})^c$ and $(\lambda^{\gamma} \times \Theta^{\gamma})$ are anti fuzzy HX rings of a $\vartheta\times\vartheta$.

Proof

Given $\psi_1 \times \psi_2$ is an intuitionistic fuzzy HX ring of $\vartheta\times\vartheta$, then

(i) $(\lambda^{\mu} \times \Theta^{\mu})((A,B) - (C,D)) \geq \min\{(\lambda^{\mu} \times \Theta^{\mu})(A,B), (\lambda^{\mu} \times \Theta^{\mu})(C,D)\}$

(ii) $(\lambda^{\mu} \times \Theta^{\mu})((A,B) . (C,D)) \geq \min\{(\lambda^{\mu} \times \Theta^{\mu})(A,B), (\lambda^{\mu} \times \Theta^{\mu})(C,D)\}$

(iii) $(\lambda^{\gamma} \times \Theta^{\gamma})((A,B) - (C,D)) \leq \max\{(\lambda^{\gamma} \times \Theta^{\gamma})(A,B), (\lambda^{\gamma} \times \Theta^{\gamma})(C,D)\}$

(iv) $(\lambda^{\gamma} \times \Theta^{\gamma})((A,B) . (C,D)) \leq \max\{(\lambda^{\gamma} \times \Theta^{\gamma})(A,B), (\lambda^{\gamma} \times \Theta^{\gamma})(C,D)\}$

From (iii) and (iv) it is clear that $(\lambda^{\gamma} \times \Theta^{\gamma})$ is an anti fuzzy HX subring of $\vartheta\times\vartheta$.

Now,

$$\begin{aligned}(\lambda^{\mu} \times \Theta^{\mu})^c((A,B) - (C,D)) &= 1 - (\lambda^{\mu} \times \Theta^{\mu})((A,B) - (C,D)) \\ &\leq 1 - \min\{(\lambda^{\mu} \times \Theta^{\mu})(A,B), (\lambda^{\mu} \times \Theta^{\mu})(C,D)\} \\ &= 1 - \min\{1 - (\lambda^{\mu} \times \Theta^{\mu})^c(A,B), 1 - (\lambda^{\mu} \times \Theta^{\mu})^c(C,D)\} \\ &= \max\{(\lambda^{\mu} \times \Theta^{\mu})^c(A,B), (\lambda^{\mu} \times \Theta^{\mu})^c(C,D)\}\end{aligned}$$

Hence, $(\lambda^{\mu} \times \Theta^{\mu})^c((A,B) - (C,D)) \leq \max\{(\lambda^{\mu} \times \Theta^{\mu})^c(A,B), (\lambda^{\mu} \times \Theta^{\mu})^c(C,D)\}$

Also
$$\begin{aligned}(\lambda^{\mu} \times \Theta^{\mu})^c((A,B) . (C,D)) &= 1 - (\lambda^{\mu} \times \Theta^{\mu})((A,B) . (C,D)) \\ &\leq 1 - \min\{(\lambda^{\mu} \times \Theta^{\mu})(A,B), (\lambda^{\mu} \times \Theta^{\mu})(C,D)\} \\ &= 1 - \min\{1 - (\lambda^{\mu} \times \Theta^{\mu})^c(A,B), 1 - (\lambda^{\mu} \times \Theta^{\mu})^c(C,D)\} \\ &= \max\{(\lambda^{\mu} \times \Theta^{\mu})^c(A,B), (\lambda^{\mu} \times \Theta^{\mu})^c(C,D)\}\end{aligned}$$

$$(\lambda^{\mu} \times \Theta^{\mu})^c((A,B) . (C,D)) \leq \max\{(\lambda^{\mu} \times \Theta^{\mu})^c(A,B), (\lambda^{\mu} \times \Theta^{\mu})^c(C,D)\}$$

Hence, $(\lambda^{\mu} \times \Theta^{\mu})^c$ and $(\lambda^{\gamma} \times \Theta^{\gamma})$ are anti fuzzy HX rings of a HX ring ϑ.

Conversely, Let $(\lambda^{\mu} \times \Theta^{\mu})^c$ and $(\lambda^{\gamma} \times \Theta^{\gamma})$ are anti fuzzy HX rings of a HX ring ϑ.
To prove that $\psi_1 \times \psi_2 = \{A\times A, \lambda^{\mu} \times \Theta^{\mu}, \lambda^{\gamma} \times \Theta^{\gamma}\}$ be a intuitionistic fuzzy HX ring of $\vartheta x \vartheta$
It is clear that

$$(\lambda^{\gamma} \times \Theta^{\gamma})((A,B) - (C,D)) \leq \max\{(\lambda^{\gamma} \times \Theta^{\gamma})(A,B), (\lambda^{\gamma} \times \Theta^{\gamma})(C,D)\}$$
$$(\lambda^{\gamma} \times \Theta^{\gamma})((A,B) . (C,D)) \leq \max\{(\lambda^{\gamma} \times \Theta^{\gamma})(A,B), (\lambda^{\gamma} \times \Theta^{\gamma})(C,D)\}$$

Now

$$(\lambda^{\mu} \times \Theta^{\mu})^c((A,B) - (C,D)) \leq \max\{(\lambda^{\mu} \times \Theta^{\mu})^c(A,B), (\lambda^{\mu} \times \Theta^{\mu})^c(C,D)\}$$
$$1 - (\lambda^{\mu} \times \Theta^{\mu})((A,B) - (C,D)) = \max\{1 - (\lambda^{\mu} \times \Theta^{\mu})(A,B), 1 - (\lambda^{\mu} \times \Theta^{\mu})(C,D)\}$$
$$1 - (\lambda^{\mu} \times \Theta^{\mu})((A,B) - (C,D)) = 1 - \min\{(\lambda^{\mu} \times \Theta^{\mu})(A,B), (\lambda^{\mu} \times \Theta^{\mu})(C,D)\}$$
$$(\lambda^{\mu} \times \Theta^{\mu})((A,B) - (C,D)) \geq \min\{(\lambda^{\mu} \times \Theta^{\mu})(A,B), (\lambda^{\mu} \times \Theta^{\mu})(C,D)\}$$

Also

$$(\lambda^{\mu} \times \Theta^{\mu})^c((A,B) . (C,D)) \leq \max\{(\lambda^{\mu} \times \Theta^{\mu})^c(A,B), (\lambda^{\mu} \times \Theta^{\mu})^c(C,D)\}$$
$$1 - (\lambda^{\mu} \times \Theta^{\mu})((A,B) . (C,D)) = \max\{1 - (\lambda^{\mu} \times \Theta^{\mu})(A,B), 1 - (\lambda^{\mu} \times \Theta^{\mu})(C,D)\}$$
$$1 - (\lambda^{\mu} \times \Theta^{\mu})((A,B) . (C,D)) = 1 - \min\{(\lambda^{\mu} \times \Theta^{\mu})(A,B), (\lambda^{\mu} \times \Theta^{\mu})(C,D)\}$$
$$(\lambda^{\mu} \times \Theta^{\mu})((A,B) . (C,D)) \geq \min\{(\lambda^{\mu} \times \Theta^{\mu})(A,B), (\lambda^{\mu} \times \Theta^{\mu})(C,D)\}$$

Thus

$$(\lambda^{\mu} \times \Theta^{\mu})((A,B) - (C,D)) \geq \min\{(\lambda^{\mu} \times \Theta^{\mu})(A,B), (\lambda^{\mu} \times \Theta^{\mu})(C,D)\},$$
$$(\lambda^{\mu} \times \Theta^{\mu})((A,B) . (C,D)) \geq \min\{(\lambda^{\mu} \times \Theta^{\mu})(A,B), (\lambda^{\mu} \times \Theta^{\mu})(C,D)\},$$
$$(\lambda^{\gamma} \times \Theta^{\gamma})((A,B) - (C,D)) \leq \max\{(\lambda^{\gamma} \times \Theta^{\gamma})(A,B), (\lambda^{\gamma} \times \Theta^{\gamma})(C,D)\},$$
$$(\lambda^{\gamma} \times \Theta^{\gamma})((A,B) . (C,D)) \leq \max\{(\lambda^{\gamma} \times \Theta^{\gamma})(A,B), (\lambda^{\gamma} \times \Theta^{\gamma})(C,D)\}.$$

Hence $\psi_1 \times \psi_2 = \{A\times A, \lambda^{\mu} \times \Theta^{\mu}, \lambda^{\gamma} \times \Theta^{\gamma}\}$ be an intuitionistic fuzzy HX ring of $\vartheta x \vartheta$

IV. Conclusion

In this paper we introduce the concept of direct product in an intuitionistic fuzzy HX ring and discuss the basic results . Further investigation may be in intuitionistic fuzzy HX ideals on HX ring. which will give a new horizon in the further study.

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