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Bipolar Anti Fuzzy Hx Ring

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Abstract : *In this paper, we define the notion of bipolar anti fuzzy HX subring of a HX ring and some of their related properties are investigated. We discuss the concept of an image, pre-image of a bipolar anti fuzzy HX subring and homomorphic, anti homomorphic properties of a bipolar anti fuzzy HX subring of a HX ring are discussed.*

Keywords: *intuitionistic fuzzy set, fuzzy HX ring, bipolar anti fuzzy HX subring, homomorphism and anti homomorphism of a bipolar anti fuzzy HX subring, image and pre-image of a bipolar anti fuzzy HX subring.*

Introduction

In 1965, Zadeh [7] introduced the concept of fuzzy subset μ of a set X . Bipolar valued fuzzy set, which was introduced by K.M.Lee [4] are an extension of fuzzy sets whose membership degree range is enlarged from the interval [0, 1] to [−1, 1]. In a bipolar-valued fuzzy set, the membership degree 0 means that the elements are irrelevant to the corresponding property, the membership degree (0, 1] indicates that elements somewhat satisfy the property and the membership degree [−1, 0) indicates that elements somewhat satisfy the implicit counter-property. R.Muthuraj et.al[6]., introduced the concept of Bipolar fuzzy and anti fuzzy HX subgroup. In 1988, Professor Li Hong Xing [3] proposed the concept of HX ring and derived some of its properties, then Professor Zhong [2] gave the structures of HX ring on a class of ring. R.Muthuraj et.al[5]., introduced the concept of fuzzy HX ring. In this paper we define a new algebraic structure of a bipolar anti fuzzy HX subring of a HX ring and investigate some related properties. Also we introduce the image and pre-image of a bipolar anti fuzzy HX subring and discuss some of its properties.

Preliminary

In this section, we site the fundamental definitions that will be used in the sequel. Throughout this paper, R = (R ,+, ·) is a Ring, e is the additive identity element of R and xy, we mean x.y

2.1 Definition

Let R be a ring. In 2^R - {ϕ}, a non-empty set $\vartheta \subset 2^R - \{\phi\}$ with two binary operation '+' and '.' is said to be a HX ring on R if ϑ is a ring with respect to the algebraic operation defined by i.A + B = {a + b / a ∈ A and b ∈ B} , which its null element is denoted by Q , and the negative element of A is denoted by – A.

ii. $AB = \{ab \,/\, a \in A \text{ and } b \in B\}$,

iii. $A (B + C) = AB + AC$ and $(B + C) A = BA + CA$.

3 Bipolar anti fuzzy HX subring of a HX ring

In this section we define the concept of a bipolar anti fuzzy HX subring of a HX ring and discuss some related results.

3.1 Definition

Let R be a ring. Let $\mu = \{ \langle x, \mu^+(x), \mu^-(x)\rangle \,/\, x\in R \}$ be a bipolar fuzzy set defined on a ring R, where $\mu^+ : R\to[0,1]$, $\mu^- : R\to[-1,0]$. Let $\Re \subset 2^R - \{\phi\}$ be a HX ring. A bipolar fuzzy subset $\lambda_\mu = \{ \langle A, \lambda^+_\mu (A), \lambda^-_\mu (A)\rangle \,/\, A\in\Re \}$ of $\Re$ is called a bipolar anti fuzzy HX subring of $\Re$ or a bipolar anti fuzzy subring induced by μ if the following conditions are satisfied. For all A,B $\in\Re$,

(i). $\lambda^+_\mu (A - B) \le \max \{ \lambda^+_\mu (A), \lambda^+_\mu (B) \}$, (ii). $\lambda^+_\mu (AB) \le \max \{ \lambda^+_\mu (A), \lambda^+_\mu (B) \}$,

(iii). $\lambda^-_\mu (A- B) \ge \min \{ \lambda^-_\mu (A) , \lambda^-_\mu (B) \}$, (iv) $\lambda^-_\mu (AB) \ge \min \{ \lambda^-_\mu (A) , \lambda^-_\mu (B)$.

where $\lambda^+_\mu (A) = \min\{ \mu^+(x) \,/\,$ for all $x\in A \subseteq R \}$ and $\lambda^-_\mu (A) = \max\{\mu^-(x)\rangle \,/\,$ for all $x\in A \subseteq R \}$.

3.2 Theorem

If $\mu = \{ \langle x, \mu^+(x), \mu^-(x)\rangle \,/\, x\in R \}$ is a bipolar anti fuzzy subring of a ring R then the bipolar fuzzy subset λ_μ is a bipolar anti fuzzy HX subring of a HX ring $\Re$.

Proof: It is clear

3.3 Theorem

Let G and H be any two bipolar fuzzy sets on R. Let γ_G and λ_H be any two bipolar anti fuzzy HX subrings of a HX ring $\Re$ then their union, $\gamma_G \cup \lambda_H$ is also a bipolar anti fuzzy HX subring of a HX ring $\Re$.

Proof It is clear.

3.4 Theorem

Let G and H be any two bipolar fuzzy sets on R. Let γ_G and λ_H be any two bipolar anti fuzzy HX subrings of a HX ring $\Re$ then their intersection, $\gamma_G \cap \lambda_H$ is also a bipolar anti fuzzy HX subring of a HX ring $\Re$.

Proof Let $G = \{ \langle x, \alpha^+(x), \alpha^-(x)\rangle \,/\, x\in R \}$ and $H = \{ \langle x, \beta^+(x), \beta^-(x)\rangle \,/\, x\in R \}$ be any two bipolar fuzzy sets defined on a ring R.Then, $\gamma_G = \{ \langle A, \gamma^+_\alpha(A), \gamma^-_\alpha (A)\rangle \,/\, A\in\Re \}$ and $\lambda_H = \{ \langle A, \lambda^+_\beta(A), \lambda^-_\beta (A)\rangle \,/\, A\in\Re \}$ be any two bipolar anti fuzzy HX subrings of a HX ring $\Re$.Then, $\gamma_G \cap \lambda_H = \{ \langle A, (\gamma^+_\alpha \cap \lambda^+_\beta) (A), (\gamma^-_\alpha \cup \lambda^-_\beta)(A)\rangle \,/\, A\in\Re \}$

Let A,B $\in \Re$

$$
\begin{aligned}
\text{i. } (\gamma^+_\alpha \cap \lambda^+_\beta) (A-B) &= \min \{ \gamma^+_\alpha (A-B) , \lambda^+_\beta (A-B)\} \\
&\le \min \{\max \{\gamma^+_\alpha (A), \gamma^+_\alpha (B)\},\max\{ \lambda^+_\beta (A), \lambda^+_\beta (B)\}\} \\
&= \max \{\min \{\gamma^+_\alpha (A), \lambda^+_\beta (A)\}, \min \{\gamma^+_\alpha (B), \lambda^+_\beta (B)\}\} \\
&= \max \{ (\gamma^+_\alpha \cap \lambda^+_\beta)(A) , (\gamma^+_\alpha \cap \lambda^+_\beta)(B)\} \\
(\gamma^+_\alpha \cap \lambda^+_\beta) (A-B) &\le \max \{ (\gamma^+_\alpha \cup \lambda^+_\beta)(A) , (\gamma^+_\alpha \cup \lambda^+_\beta)(B)\}. \\
\text{ii. } (\gamma^+_\alpha \cap \lambda^+_\beta) (AB) &= \min \{ \gamma^+_\alpha (AB) , \lambda^+_\beta (AB)\} \\
&\le \min \{\max \{\gamma^+_\alpha (A), \gamma^+_\alpha (B)\},\max\{ \lambda^+_\beta (A), \lambda^+_\beta (B)\}\} \\
&= \max \{\min \{\gamma^+_\alpha (A), \lambda^+_\beta (A)\}, \min\{\gamma^+_\alpha (B), \lambda^+_\beta (B)\}\} \\
&= \max \{ (\gamma^+_\alpha \cap \lambda^+_\beta)(A) , (\gamma^+_\alpha \cap \lambda^+_\beta)(B)\} \\
(\gamma^+_\alpha \cap \lambda^+_\beta) (AB) &\le \max \{ (\gamma^+_\alpha \cap \lambda^+_\beta)(A) , (\gamma^+_\alpha \cap \lambda^+_\beta)(B)\}.
\end{aligned}
$$

iii. $(\gamma^-_\alpha \cup \lambda^-_\beta)(A-B) = \max\{\gamma^-_\alpha(A-B), \lambda^-_\beta(A-B)\}$
$\geq \max\{\min\{\gamma^-_\alpha(A), \gamma^-_\alpha(B)\}, \min\{\lambda^-_\beta(A), \lambda^-_\beta(B)\}\}$
$= \min\{\max\{\gamma^-_\alpha(A), \lambda^-_\beta(A)\}, \max\{\gamma^-_\alpha(B), \lambda^-_\beta(B)\}\}$
$= \min\{(\gamma^-_\alpha \cup \lambda^-_\beta)(A), (\gamma^-_\alpha \cup \lambda^-_\beta)(B)\}$
$(\gamma^-_\alpha \cup \lambda^-_\beta)(A-B) \geq \min\{(\gamma^-_\alpha \cup \lambda^-_\beta)(A), (\gamma^-_\alpha \cup \lambda^-_\beta)(B)\}$.

iv. $(\gamma^-_\alpha \cup \lambda^-_\beta)(AB) = \max\{\gamma^-_\alpha(AB), \lambda^-_\beta(AB)\}$
$\geq \max\{\min\{\gamma^-_\alpha(A), \gamma^-_\alpha(B)\}, \min\{\lambda^-_\beta(A), \lambda^-_\beta(B)\}\}$
$= \min\{\max\{\gamma^-_\alpha(A), \lambda^-_\beta(A)\}, \max\{\gamma^-_\alpha(B), \lambda^-_\beta(B)\}\}$
$= \min\{(\gamma^-_\alpha \cup \lambda^-_\beta)(A), (\gamma^-_\alpha \cup \lambda^-_\beta)(B)\}$
$(\gamma^-_\alpha \cup \lambda^-_\beta)(AB) \geq \min\{(\gamma^-_\alpha \cup \lambda^-_\beta)(A), (\gamma^-_\alpha \cup \lambda^-_\beta)(B)\}$.

Hence, $\gamma_G \cap \lambda_H$ is a bipolar anti fuzzy HX subring of a HX ring $\Re$.

3.5 Definition

Let $G = \{\langle x, \alpha^+(x), \alpha^-(x)\rangle / x\in R\}$ and $H = \{\langle x, \beta^+(x), \beta^-(x)\rangle / x\in R\}$ be any two bipolar fuzzy sets defined on a ring R. Let $\Re_1 \subset 2^{R_1} - \{\phi\}$ and $\Re_2 \subset 2^{R_2} - \{\phi\}$ be any two HX rings. Let $\gamma_G = \{\langle A, \gamma^+_\alpha(A), \gamma^-_\alpha(A)\rangle / A\in\Re\}$ and $\lambda_H = \{\langle A, \lambda^+_\beta(A), \lambda^-_\beta(A)\rangle / A\in\Re\}$ be any two bipolar fuzzy subsets of a HX ring $\Re$, then the anti-product of γ_G and λ_H is defined as

$$(\gamma_G \times \lambda_H) = \{\langle (A,B), (\gamma^+_\alpha \cup \lambda^+_\beta)(A,B), (\gamma^-_\alpha \cap \lambda^-_\beta)(A,B)\rangle / (A,B) \in \Re_1 \times \Re_2\},$$

where, $(\gamma^+_\alpha \cup \lambda^+_\beta)(A,B) = \max\{\gamma^+_\alpha(A), \lambda^+_\beta(B)\}$, for all $(A,B) \in \Re_1 \times \Re_2$,
$(\gamma^-_\alpha \cap \lambda^-_\beta)(A,B) = \min\{\gamma^-_\alpha(A), \lambda^-_\beta(B)\}$, for all $(A,B) \in \Re_1 \times \Re_2$.

3.6 Theorem

Let G and H be any two bipolar fuzzy sets of R_1 and R_2 respectively. Let $\Re_1 \subset 2^{R_1} - \{\phi\}$ and $\Re_2 \subset 2^{R_2} - \{\phi\}$ be any two HX rings. If γ_G and λ_H are any two bipolar anti fuzzy HX subrings of $\Re_1$ and $\Re_2$ respectively then, $\gamma_G \times \lambda_H$ is also a bipolar anti fuzzy HX subring of a HX ring $\Re_1 \times \Re_2$.

Proof It is clear.

3.7 Theorem

Let $H = \{\langle x, \beta^+(x), \beta^-(x)\rangle / x\in R\}$ be a bipolar fuzzy set defined on R. Let $\lambda_H = \{\langle A, \lambda^+_\beta(A), \lambda^-_\beta(A)\rangle / A\in\Re\}$ be a bipolar fuzzy HX subring of $\Re$ if and only if $(\lambda_H)^c$ is a bipolar anti fuzzy HX subring of $\Re$.

Proof Let λ_H be a bipolar fuzzy HX subring of $\Re$.

Let $A,B \in \Re$

i. $\lambda^+_\beta(A-B) \geq \min\{\lambda^+_\beta(A), \lambda^+_\beta(B)\}$
$\Leftrightarrow 1 - \lambda^+_\beta(A-B) \leq 1 - \min\{\lambda^+_\beta(A), \lambda^+_\beta(B)\}$
$\Leftrightarrow (\lambda^+_\beta)^c(A-B) \leq \max\{(1-\lambda^+_\beta(A)), (1-\lambda^+_\beta(B))\}$
$\Leftrightarrow (\lambda^+_\beta)^c(A-B) \leq \max\{(\lambda^+_\beta)^c(A), (\lambda^+_\beta)^c(B)\}$.

ii. $\lambda^+_\beta(AB) \geq \min\{\lambda^+_\beta(A), \lambda^+_\beta(B)\}$
$\Leftrightarrow 1 - \lambda^+_\beta(AB) \leq 1 - \min\{\lambda^+_\beta(A), \lambda^+_\beta(B)\}$
$\Leftrightarrow (\lambda^+_\beta)^c(AB) \leq \max\{(1-\lambda^+_\beta(A)), (1-\lambda^+_\beta(B))\}$
$\Leftrightarrow (\lambda^+_\beta)^c(AB) \leq \max\{(\lambda^+_\beta)^c(A), (\lambda^+_\beta)^c(B)\}$.

iii. $\lambda^-_\beta(A-B) \leq \max\{\lambda^-_\beta(A), \lambda^-_\beta(B)\}$
$\Leftrightarrow -1 - \lambda^-_\beta(A-B) \geq -1 - \max\{\lambda^-_\beta(A), \lambda^-_\beta(B)\}$
$\Leftrightarrow (\lambda^-_\beta)^c(A-B) \geq \min\{(-1-\lambda^-_\beta(A)), (-1-\lambda^-_\beta(B))\}$
$\Leftrightarrow (\lambda^-_\beta)^c(A-B) \geq \min\{(\lambda^-_\beta)^c(A), (\lambda^-_\beta)^c(B)\}$.

iv. $\lambda^{-}_{\beta}(AB) \leq \max\{ \lambda^{-}_{\beta}(A), \lambda^{-}_{\beta}(B) \}$

$\Leftrightarrow -1-\lambda^{-}_{\beta}(AB) \geq -1-\max\{ \lambda^{-}_{\beta}(A), \lambda^{-}_{\beta}(B)\}$

$\Leftrightarrow (\lambda^{-}_{\beta})^{c}(AB) \geq \min\{ (-1-\lambda^{-}_{\beta}(A)),(-1-\lambda^{-}_{\beta}(B))\}$

$\Leftrightarrow (\lambda^{-}_{\beta})^{c}(AB) \geq \min\{ (\lambda^{-}_{\beta})^{c}(A), (\lambda^{-}_{\beta})^{c}(B)\}$.

Hence, $(\lambda^{\mu})^{c}$ is a bipolar anti fuzzy HX subring of $\Re$.

4 Homomorphism and anti homomorphism

4.1 Definition

Let R_1 and R_2 be any two rings. Let $\Re_1 \subset 2^{R_1} - \{\phi\}$ and $\Re_2 \subset 2^{R_2} - \{\phi\}$ be any two HX rings defined on R_1 and R_2 respectively. Let G and H be any two bipolar fuzzy subsets in R_1 and R_2 respectively. Let γ_G and λ_H be any two bipolar anti fuzzy HX subrings of HX rings $\Re_1$ and $\Re_2$ respectively induced by G and H. Let f: $\Re_1 \rightarrow \Re_2$ be a mapping then the image of λ_G denoted as f (λ_G) is a bipolar fuzzy subset of $\Re_2$ and is defined as for each $Y \in \Re_2$,

$$(f(\gamma^{+}_{\alpha}))(Y) = \begin{cases} \min\{\gamma^{+}_{\alpha}(X) : X \in f^{-1}(Y)\} & , \text{ if } f^{-1}(Y) \neq \phi \\ 1 & , \text{ otherwise} \end{cases}$$

$$(f(\gamma^{-}_{\alpha}))(Y) = \begin{cases} \min\{\gamma^{-}_{\alpha}(X) : X \in f^{-1}(Y)\} & , \text{ if } f^{-1}(Y) \neq \phi \\ -1 & , \text{ otherwise} \end{cases}$$

Also the pre-image of λ_H denoted as $f^{-1}(\lambda_H)$ under f is a bipolar fuzzy subset of $\Re_1$ defined as for each $X \in \Re_1$, $(f^{-1}(\lambda^{+}_{\beta}))(X) = \lambda^{+}_{\beta}(f(X))$ and $(f^{-1}(\lambda^{-}_{\beta}))(X) = \lambda^{-}_{\beta}(f(X))$.

4.2 Theorem

Let $\Re_1$ and $\Re_2$ be any two HX rings on the rings R_1 and R_2 respectively. Let f : $\Re_1 \rightarrow \Re_2$ be a homomorphism onto HX rings. Let G be a bipolar fuzzy subset of R_1. Let γ_G be a bipolar anti fuzzy HX subring of $\Re_1$ then f (γ_G) is a bipolar anti fuzzy HX subring of $\Re_2$, if γ_G has a infimum property and γ_G is f-invariant.

Proof Let G = { $\langle x, \alpha^{+}(x), \alpha^{-}(x)\rangle$ / $x \in R_1$ } be a bipolar fuzzy set defined on a ring R_1.
Then, γ_G = { $\langle X, \gamma^{+}_{\alpha}(X), \gamma^{-}_{\alpha}(X)\rangle$ / $X \in \Re_1$ } be a bipolar anti fuzzy HX subring of a HX ring $\Re_1$.
Then, $f(\gamma_G)$ = { $\langle f(X), f(\gamma^{+}_{\alpha})(f(X)), f(\gamma^{-}_{\alpha})(f(X))\rangle$ / $X \in \Re_1$ }.There exist X, Y $\in \Re_1$ such that f(X) , f(Y) $\in \Re_2$,

i.
$$\begin{aligned} (f(\gamma^{+}_{\alpha}))(f(X)-f(Y)) &= (f(\gamma^{+}_{\alpha}))(f(X-Y)) \\ &= \gamma^{+}_{\alpha}(X-Y) \\ &\leq \max\{\gamma^{+}_{\alpha}(X), \gamma^{+}_{\alpha}(Y)\} \\ &= \max\{(f(\gamma^{+}_{\alpha}))(f(X)), (f(\gamma^{+}_{\alpha}))(f(Y))\} \\ (f(\gamma^{+}_{\alpha}))(f(X)-f(Y)) &\leq \max\{(f(\gamma^{+}_{\alpha}))(f(X)), (f(\gamma^{+}_{\alpha}))(f(Y))\} \end{aligned}$$

ii.
$$\begin{aligned} (f(\gamma^{+}_{\alpha}))(f(X)f(Y)) &= (f(\gamma^{+}_{\alpha}))(f(XY)) \\ &= \gamma^{+}_{\alpha}(XY) \\ &\leq \max\{\gamma^{+}_{\alpha}(X), \gamma^{+}_{\alpha}(Y)\} \\ &= \max\{(f(\gamma^{+}_{\alpha}))(f(X)), (f(\gamma^{+}_{\alpha}))(f(Y))\} \\ (f(\gamma^{+}_{\alpha}))(f(X)f(Y)) &\leq \max\{(f(\gamma^{+}_{\alpha}))(f(X)), (f(\gamma^{+}_{\alpha}))(f(Y))\}. \end{aligned}$$

iii. $(f(\gamma^-_\alpha))(f(X)-f(Y)) = (f(\gamma^-_\alpha))(f(X-Y))$
$= \gamma^-_\alpha(X-Y)$
$\geq \min\{\gamma^-_\alpha(X), \gamma^-_\alpha(Y)\}$
$= \min\{(f(\gamma^-_\alpha))(f(X)), (f(\gamma^-_\alpha))(f(Y))\}$
$(f(\gamma^-_\alpha))(f(X)-f(Y)) \geq \min\{(f(\gamma^-_\alpha))(f(X)), (f(\gamma^-_\alpha))(f(Y))\}$

iv. $(f(\gamma^-_\alpha))(f(X)f(Y)) = (f(\gamma^-_\alpha))(f(XY))$
$= \gamma^-_\alpha(XY)$
$\geq \min\{\gamma^-_\alpha(X), \gamma^-_\alpha(Y)\}$
$= \min\{(f(\gamma^-_\alpha))(f(X)), (f(\gamma^-_\alpha))(f(Y))\}$
$(f(\gamma^-_\alpha))(f(X)f(Y)) \geq \min\{(f(\gamma^-_\alpha))(f(X)), (f(\gamma^-_\alpha))(f(Y))\}$.

Hence, $f(\gamma_G)$ is a bipolar anti fuzzy HX subring of $\mathfrak{R}_2$.

4.3 Theorem

Let $\mathfrak{R}_1$ and $\mathfrak{R}_2$ be any two HX rings on R_1 and R_2 respectively. Let $f : \mathfrak{R}_1 \rightarrow \mathfrak{R}_2$ be a homomorphism on HX rings. Let H be a bipolar fuzzy subset of R_2. Let λ_H be a bipolar anti fuzzy HX subring of $\mathfrak{R}_2$, then $f^{-1}(\lambda_H)$ is a bipolar anti fuzzy HX subring of $\mathfrak{R}_1$.

Proof Let $H = \{ \langle y, \beta^+(y), \beta^-(y)\rangle / y \in R_2 \}$ be a bipolar fuzzy set defined on R_2. Let $\lambda_H = \{ \langle Y, \lambda^+_\beta(Y), \lambda^-_\beta(Y)\rangle / Y \in \mathfrak{R}_2 \}$ be a bipolar fuzzy HX subring of $\mathfrak{R}_2$ Then, $f^{-1}(\lambda_H) = \{ \langle X, f^{-1}(\lambda^+_\beta)(X), f^{-1}(\lambda^-_\beta)(X)\rangle / X \in \mathfrak{R}_1 \}$.For any $X,Y \in \mathfrak{R}_1$, $f(X), f(Y) \in \mathfrak{R}_2$,

i. $(f^{-1}(\lambda^+_\beta))(X-Y) = \lambda^+_\beta(f(X-Y))$
$= \lambda^+_\beta(f(X)-f(Y))$
$\leq \max\{\lambda^+_\beta(f(X)), \lambda^+_\beta(f(Y))\}$
$= \max\{(f^{-1}(\lambda^+_\beta))(X), (f^{-1}(\lambda^+_\beta))(Y)\}$
$(f^{-1}(\lambda^+_\beta))(X-Y) \leq \max\{(f^{-1}(\lambda^+_\beta))(X), (f^{-1}(\lambda^+_\beta))(Y)\}$.

ii. $(f^{-1}(\lambda^+_\beta))(XY) = \lambda^+_\beta(f(XY))$
$= \lambda^+_\beta(f(X)f(Y))$
$\leq \max\{\lambda^+_\beta(f(X)), \lambda^+_\beta(f(Y))\}$
$= \max\{(f^{-1}(\lambda^+_\beta))(X), (f^{-1}(\lambda^+_\beta))(Y)\}$
$(f^{-1}(\lambda^+_\beta))(XY) \leq \max\{(f^{-1}(\lambda^+_\beta))(X), (f^{-1}(\lambda^+_\beta))(Y)\}$

iii. $(f^{-1}(\lambda^-_\beta))(X-Y) = \lambda^-_\beta(f(X-Y))$
$= \lambda^-_\beta(f(X)-f(Y))$
$\geq \min\{\lambda^-_\beta(f(X)), \lambda^-_\beta(f(Y))\}$
$= \min\{(f^{-1}(\lambda^-_\beta))(X), (f^{-1}(\lambda^-_\beta))(Y)\}$
$(f^{-1}(\lambda^-_\beta))(X-Y) \geq \min\{(f^{-1}(\lambda^-_\beta))(X), (f^{-1}(\lambda^-_\beta))(Y)\}$.

iv. $(f^{-1}(\lambda^-_\beta))(XY) = \lambda^-_\beta(f(XY))$
$= \lambda^-_\beta(f(X)f(Y))$
$\geq \min\{\lambda^-_\beta(f(X)), \lambda^-_\beta(f(Y))\}$
$= \min\{(f^{-1}(\lambda^-_\beta))(X), (f^{-1}(\lambda^-_\beta))(Y)\}$
$(f^{-1}(\lambda^-_\beta))(XY) \geq \min\{(f^{-1}(\lambda^-_\beta))(X), (f^{-1}(\lambda^-_\beta))(Y)\}$.

Hence, $f^{-1}(\lambda_H)$ is a bipolar anti fuzzy HX subring of $\mathfrak{R}_1$.

4.4 Theorem

Let $\Re_1$ and $\Re_2$ be any two HX rings on the rings R_1 and R_2 respectively. Let f : $\Re_1 \rightarrow \Re_2$ be an anti homomorphism on HX rings. Let G be a bipolar fuzzy subset of R_1. Let γ_G be a bipolar anti fuzzy HX subring of $\Re_1$ then f (γ_G) is a bipolar anti fuzzy HX subring of $\Re_2$, if γ_G has a infimum property and γ_G is f-invariant.

Proof Let G = { $\langle x, \alpha^+(x), \alpha^-(x)\rangle$ / $x \in R_1$ } be a bipolar fuzzy set defined on a ring R_1.

Then, γ_G = { $\langle X, \gamma^+_\alpha(X), \gamma^-_\alpha(X)\rangle$ / $X \in \Re_1$ } be a bipolar anti fuzzy HX subring of a HX ring $\Re_1$. Then, $f(\gamma_G)$ = { $\langle f(X), f(\gamma^+_\alpha)(f(X)), f(\gamma^-_\alpha)(f(X))\rangle$ / $X \in \Re_1$}.There exist X, Y $\in \Re_1$ such that f(X) , f(Y) $\in \Re_2$,

i. $$\begin{aligned}(f(\gamma^+_\alpha))(f(X)-f(Y)) &= (f(\gamma^+_\alpha))(f(X-Y)),\\ &= \gamma^+_\alpha(X-Y)\\ &\leq \max\{\gamma^+_\alpha(X), \gamma^+_\alpha(Y)\}\\ &= \max\{(f(\gamma^+_\alpha))(f(X)), (f(\gamma^+_\alpha))(f(Y))\}\\ (f(\gamma^+_\alpha))(f(X)-f(Y)) &\leq \max\{(f(\gamma^+_\alpha))(f(X)), (f(\gamma^+_\alpha))(f(Y))\}\end{aligned}$$

ii. $$\begin{aligned}(f(\gamma^+_\alpha))(f(X)f(Y)) &= (f(\gamma^+_\alpha))(f(XY))\\ &= \gamma^+_\alpha(YX)\\ &\leq \max\{\gamma^+_\alpha(Y), \gamma^+_\alpha(X)\}\\ &\leq \max\{\gamma^+_\alpha(X), \gamma^+_\alpha(Y)\}\\ &= \max\{(f(\gamma^+_\alpha))(f(X)), (f(\gamma^+_\alpha))(f(Y))\}\\ (f(\gamma^+_\alpha))(f(X)f(Y)) &\leq \max\{(f(\gamma^+_\alpha))(f(X)), (f(\gamma^+_\alpha))(f(Y))\}.\end{aligned}$$

iii. $$\begin{aligned}(f(\gamma^-_\alpha))(f(X)-f(Y)) &= (f(\gamma^-_\alpha))(f(X-Y)),\\ &= \gamma^-_\alpha(X-Y)\\ &\geq \min\{\gamma^-_\alpha(X), \gamma^-_\alpha(Y)\}\\ &= \min\{(f(\gamma^-_\alpha))(f(X)), (f(\gamma^-_\alpha))(f(Y))\}\\ (f(\gamma^-_\alpha))(f(X)-f(Y)) &\geq \min\{(f(\gamma^-_\alpha))(f(X)), (f(\gamma^-_\alpha))(f(Y))\}\end{aligned}$$

iv. $$\begin{aligned}(f(\gamma^-_\alpha))(f(X)f(Y)) &= (f(\gamma^-_\alpha))(f(XY))\\ &= \gamma^-_\alpha(YX)\\ &\geq \min\{\gamma^-_\alpha(Y), \gamma^-_\alpha(X)\}\\ &= \min\{\gamma^-_\alpha(X), \gamma^-_\alpha(Y)\}\\ &= \min\{(f(\gamma^-_\alpha))(f(X)), (f(\gamma^-_\alpha))(f(Y))\}\\ (f(\gamma^-_\alpha))(f(X)f(Y)) &\geq \min\{(f(\gamma^-_\alpha))(f(X)), (f(\gamma^-_\alpha))(f(Y))\}.\end{aligned}$$

Hence, f (γ_G) is a bipolar anti fuzzy HX subring of $\Re_2$.

4.5 Theorem

Let $\Re_1$ and $\Re_2$ be any two HX rings on R_1 and R_2 respectively. Let f : $\Re_1 \rightarrow \Re_2$ be an anti homomorphism on HX rings. Let H be a bipolar fuzzy subset of R_2. Let λ_H be a bipolar anti fuzzy HX subring of $\Re_2$, then $f^{-1}(\lambda_H)$ is a bipolar anti fuzzy HX subring of $\Re_1$.

Proof Let H = { $\langle y, \beta^+(y), \beta^-(y)\rangle$ / $y \in R_2$ } be a bipolar fuzzy set defined on R_2. Let λ_H = { $\langle Y, \lambda^+_\beta(Y), \lambda^-_\beta(Y)\rangle$ / $Y \in \Re_2$ } be a bipolar fuzzy HX subring of $\Re_2$ Then, $f^{-1}(\lambda_H)$ = { $\langle X, f^{-1}(\lambda^+_\beta)(X), f^{-1}(\lambda^-_\beta)(X)\rangle$ / $X \in \Re_1$ }.For any X,Y $\in \Re_1$, f(X) , f(Y) $\in \Re_2$,

i. $$\begin{aligned}(f^{-1}(\lambda^+_\beta))(X-Y) &= \lambda^+_\beta(f(X-Y))\\ &= \lambda^+_\beta(f(X)-f(Y))\\ &\leq \max\{\lambda^+_\beta(f(X)), \lambda^+_\beta(f(Y))\}\end{aligned}$$

$$= \max\{(f^{-1}(\lambda^{+}{}_{\beta}))(X), (f^{-1}(\lambda^{+}{}_{\beta}))(Y)\}$$

$$(f^{-1}(\lambda^{+}{}_{\beta}))(X-Y) \leq \max\{(f^{-1}(\lambda^{+}{}_{\beta}))(X), (f^{-1}(\lambda^{+}{}_{\beta}))(Y)\}.$$

ii. $$(f^{-1}(\lambda^{+}{}_{\beta}))(XY) = \lambda^{+}{}_{\beta}(f(XY))$$
$$= \lambda^{+}{}_{\beta}(f(Y)\, f(X))$$
$$\leq \max\{\lambda^{+}{}_{\beta}(f(Y)), \lambda^{+}{}_{\beta}(f(X))\}$$
$$= \max\{\lambda^{+}{}_{\beta}(f(X)), \lambda^{+}{}_{\beta}(f(Y))\}$$
$$= \max\{(f^{-1}(\lambda^{+}{}_{\beta}))(X), (f^{-1}(\lambda^{+}{}_{\beta}))(Y)\}$$
$$(f^{-1}(\lambda^{+}{}_{\beta}))(XY) \leq \max\{(f^{-1}(\lambda^{+}{}_{\beta}))(X), (f^{-1}(\lambda^{+}{}_{\beta}))(Y)\}$$

iii. $$(f^{-1}(\lambda^{-}{}_{\beta}))(X-Y) = \lambda^{-}{}_{\beta}(f(X-Y))$$
$$= \lambda^{-}{}_{\beta}(f(X) - f(Y))$$
$$\geq \min\{\lambda^{-}{}_{\beta}(f(X)), \lambda^{-}{}_{\beta}(f(Y))\}$$
$$= \min\{(f^{-1}(\lambda^{-}{}_{\beta}))(X), (f^{-1}(\lambda^{-}{}_{\beta}))(Y)\}$$
$$(f^{-1}(\lambda^{-}{}_{\beta}))(X-Y) \geq \min\{(f^{-1}(\lambda^{-}{}_{\beta}))(X), (f^{-1}(\lambda^{-}{}_{\beta}))(Y)\}.$$

iv. $$(f^{-1}(\lambda^{-}{}_{\beta}))(XY) = \lambda^{-}{}_{\beta}(f(XY))$$
$$= \lambda^{-}{}_{\beta}(f(Y)\, f(X))$$
$$\geq \min\{\lambda^{-}{}_{\beta}(f(Y)), \lambda^{-}{}_{\beta}(f(X))\}$$
$$= \min\{\lambda^{-}{}_{\beta}(f(X)), \lambda^{-}{}_{\beta}(f(Y))\}$$
$$= \min\{(f^{-1}(\lambda^{-}{}_{\beta}))(X), (f^{-1}(\lambda^{-}{}_{\beta}))(Y)\}$$
$$(f^{-1}(\lambda^{-}{}_{\beta}))(XY) \geq \min\{(f^{-1}(\lambda^{-}{}_{\beta}))(X), (f^{-1}(\lambda^{-}{}_{\beta}))(Y)\}.$$

Hence, $f^{-1}(\lambda_H)$ is a bipolar anti fuzzy HX subring of $\mathfrak{R}_1$.

Conclusion

In this paper we introduce the concept of bipolar anti fuzzy HX ring and discuss the basic results on HX ring. Further investigation may be in bipolar anti fuzzy HX ideals on HX ring which will give a new horizon in the further study.

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