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# A New Type Of Closed Sets In Terms Of Grills

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***Abstract***

*In this paper, we introduce and investigate the notions of αω- border, αω-derived, αω-frontier, αω-exterior of a set using the concept of αω- open sets. Further, we shall define the αω(θ)-adherence and αω(θ)- convergence using the concept of grills and study some of their properties.*

Keywords. αω-derived, αω-border, αω-frontier, αω-exterior, Grill, αω(θ)convergence and αω(θ)-adherence of a grill, αω-closed space.

**AMS Subject Classification:** 54B05, 54 C 08

**Introduction**

Recently, the concept of $\alpha\omega$-closed sets in topological spaces was introduced by M. Parimala et al. in [6] and further more they studied their properties and its applications in [7, 8]. The idea of grill was introduced by G.Choquent [1] in 1947 and since then it has been observed in connection with many mathematical investigation such as the theories of proximity spaces, compactification etc, that grills as a tool (like filters) are extremely useful and convenient for many situations.
In 2006, M.N. Mukherjee and B.Roy [4] studied the notion of *p*-closed sets in

topological spaces in-terms of grills.

In this paper, we introduce the notions of $\alpha\omega$-derived, $\alpha\omega$-border, $\alpha\omega$- frontier and $\alpha\omega$-exterior of a set and show that some of their properties are analogous to those for open sets. Further, we shall define the $\alpha\omega(\theta)$-adherence and $\alpha\omega(\theta)$convergence of a grill and develop the concept to some extent so that the result derive here may support our subsequent deliberations.

# 1 Preliminaries

All through this paper, $(X,\tau)$ and $(Y,\sigma)$ (or simply $X$ and $Y$ ) stand for topological spaces with no separation axioms assumed, unless otherwise stated. Let $A \subseteq X$, the closure of $A$ and the interior of $A$ will be denoted by $cl(A)$ and $int(A)$

respectively.

Let $A \subseteq X$, the closure of $A$ and the interior of $A$ will be denoted by $cl(A)$ and $int(A)$ respectively. **Definition 2.1.** [1] A grill G on a topological space $X$ is defined to be a collection of non empty subsets of $X$ such that

(i) $A \in$ G and $A \subseteq B \subseteq X \Rightarrow B \in$ G and

(ii) $A,B \subseteq X$ and $A \cup B \in$ G $\Rightarrow A \in$ G or $B \in$ G.

**Definition 2.2.** A subset $A$ of a space $(X,\tau)$ is called a

1. semi-open set [2] if $A \subseteq cl(int(A))$ and a semi-closed set if $int(cl(A)) \subseteq A$,

2. $\alpha$-open set [5] if $A \subseteq int(cl(int(A)))$ and an $\alpha$-closed set if $cl(int(cl(A))) \subseteq$

$A$,

3. pre open set [3] if $A \subseteq int(cl(A))$ and pre closed set if $cl(int(A)) \subseteq A$,

4. $\delta$-open set [12] if for each $x \in A$, there exists a regular open set $G$ such that

$x \in G \subset A$ and

5. regular open set [13] if $A = int(cl(A))$ and regular closed if its complement is regular open; equivalently $A$ is regular closed if $A = cl(int(A))$.

The semi-closure (resp. $\alpha$-closure) of a subset $A$ of a space $(X,\tau)$ is the intersection of all semi-closed (resp. $\alpha$-closed) sets that contain $A$ and is denoted by $scl(A)$ (resp. $\alpha cl(A)$).

**Definition 2.3.** A subset $A$ of a space $(X,\tau)$ is called a

1. a $\omega(= g)$-*closed* set [9, 11] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is *semi-open* b

in $(X,\tau)$. The complement of *ω-closed* set is called *ω-open* set and

2. a *αω-closed* set [6] if $\omega cl(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is *α-open* in $(X,\tau)$. The complement of *αω-closed* set is called *αω-open* set.

The set of all $\alpha\omega$-open sets of $X$ will be denoted by $\alpha\omega O(X)$ and the said of all those members of $\alpha\omega O(X)$, which contain a given point $x$ of $X$ will be designated by $\alpha\omega O(x)$. The intersection of all $\alpha\omega$-closed sets in $X$, which are contained in a given set $A(\subseteq X)$ is called the $\alpha\omega$-closure of $A$, to be denoted by $\mathrm{cl}_{\alpha\omega}(A)$. It is known that for $x \in X$ and $A \subseteq X$, $x \in \alpha\omega\text{-cl}(A)$ if and only if $U \cap A$ 6= $\varphi$, for all

$U \in \alpha\omega O(x)$. Again for any set $A$ in $X$, $\alpha\omega(\theta)\text{-cl}(A)$, denoted by $\alpha\omega(\theta)\text{-cl}(A)$, is

n o defined as $\alpha\omega(\theta)\text{-cl}(A) =$ $x \in X : \alpha\omega\text{-cl}(U) \cap A$ 6= $\varphi$ for all $U \in \alpha\omega O(x)$ .

## 2 More on $\alpha\omega$-closed sets in topological spaces

**Definition 3.1.** Let $A$ be a subset of space $X$. A point $x \in X$ is said to be $\alpha\omega$-limit point of $A$ if for each $\alpha\omega$-open set $U$ containing $x$, $U \cap (A - \{x\})$ 6= $\varphi$. The set of all $\alpha\omega$-limit points of $A$ is called a $\alpha\omega$-derived set of $A$ and is denoted by $D_{\alpha\omega}(A)$.

**Theorem 3.2.** For subsets $A,B$ of a space $X$, the following statements hold:

(i) $D_{\alpha\omega}(A) \subset D(A)$ where $D(A)$ is the derived set of $A$

(ii) If $A \subset B$, then $D_{\alpha\omega}(A) \subset D_{\alpha\omega}(B)$

(iii) $D_{\alpha\omega}(A) \cup D_{\alpha\omega}(B) \subset D_{\alpha\omega}(A \cup B)$

(iv) $D_{\alpha\omega}(D_{\alpha\omega}(A)) - A \subset D_{\alpha\omega}(A)$

(v) $D_{\alpha\omega}(A \cup D_{\alpha\omega}(A)) \subset A \cup D_{\alpha\omega}(A)$

**Proof.**

(i) It suffices to observe that every open set is $\alpha\omega$- open.

(iii) It is an immediate consequence of ( ii ).

(iv) If $x \in D_{\alpha\omega}(D_{\alpha\omega}(A)) - A$ and $U$ is a $\alpha\omega$-open set containing $x$, then $U \cap (D_{\alpha\omega}(A) - \{x\})$ 6= $\varphi$. Let $y \in U \cap (D_{\alpha\omega}(A) - \{x\})$. Then since $y \in D_{\alpha\omega}(A)$

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and $y \in U$, $U \cap (A - \{y\})$ 6= $\varphi$. Let $Z \in U \cap (A - \{y\})$. Then $Z$ 6= $x$ for $Z \in A$ and $x \notin A$. Hence $U \cap (A - \{x\})$ =6 $\varphi$. Therefore $x \in D_{\alpha\omega}(A)$.

(v) Let $x \in D_{\alpha\omega}(A \cup D_{\alpha\omega}(A))$. If $x \in A$, the result is obvious. So let $x \in$

$D_{\alpha\omega}(A \cup D_{\alpha\omega}(A)) - A$, then for $\alpha\omega$-open set $U$ containing $x$, $U \cap (A \cup D_{\alpha\omega}(A) - \{x\})$ 6= $\varphi$. Thus $U \cap (A - \{x\})$ 6= $\varphi$ or $U \cap (D_{\alpha\omega}(A) - \{x\})$ 6= $\varphi$. Now it follows (iv) that $U \cap (A - \{x\})$ 6= $\varphi$. Hence $x \in D_{\alpha\omega}(A)$. Therefore, in any case $D_{\alpha\omega}(A \cup D_{\alpha\omega}(A)) \subset A \cup D_{\alpha\omega}(A)$.

In general the converse of (i) may not be true by the following Example.

**Example 3.3.** Let $X = \{a,b,c\}$ with topology $\tau = \{X,\varphi,\{a,b\}\}$. Thus $\alpha\omega O(X) = \{X,\varphi,\{a\},\{b\},\{a,b\}\}$. Take $A = \{a,b\}$, we obtain $D(A)$ does not contained in $D_{\alpha\omega}(A)$.

**Theorem 3.4.** For any subset $A$ of a space $X$, $\mathrm{cl}_{\alpha\omega}(A) = A \cup D_{\alpha\omega}(A)$.

**Proof.** Since $D_{\alpha\omega}(A) \subset \mathrm{cl}_{\alpha\omega}(A)$, $A \cup D_{\alpha\omega}(A) \subset \mathrm{cl}_{\alpha\omega}(A)$. On the other hand, let $x \in \mathrm{cl}_{\alpha\omega}(A)$. If $x \in A$, then the proof is complete. If $x \notin A$, then each $\alpha\omega$- open set $U$ containing $x$ intersects $A$ at a point distinct from $x$. Therefore $x \in D_{\alpha\omega}(A)$. Thus $\mathrm{cl}_{\alpha\omega}(A) \subset A \cup D_{\alpha\omega}(A)$ which completes the proof.

**Definition 3.5.** A point $x \in X$ is said to be a $\alpha\omega$-interior point of $A$ if there exists a $\alpha\omega$-open sets $U$ containing $x$ such that $U \subset A$. The set of all $\alpha\omega$- interior points of $A$ is said to be $\alpha\omega$-interior of $A$ and is denoted by $\mathrm{int}_{\alpha\omega}(A)$.

**Theorem 3.6.** For subset $A$,$B$ of a space $X$, the following statements hold:

(i) $\mathrm{int}_{\alpha\omega}(A)$ is the largest $\alpha\omega$-open set contained in $A$.

(ii) $A$ is $\alpha\omega$-open if and only if $A = \mathrm{int}_{\alpha\omega}(A)$.

(iii) $\mathrm{int}_{\alpha\omega}(\mathrm{int}_{\alpha\omega}(A)) = \mathrm{int}_{\alpha\omega}(A)$.

(iv) $\mathrm{int}_{\alpha\omega}(A) = A - D_{\alpha\omega}(X - A)$.

(v) $X - \mathrm{int}_{\alpha\omega}(A) = \mathrm{cl}_{\alpha\omega}(X - A)$. (vi) $X - \mathrm{cl}_{\alpha\omega}(A) = \mathrm{int}_{\alpha\omega}(X - A)$.

(vii) $A \subset B$, then $\mathrm{int}_{\alpha\omega}(A) \subset \mathrm{int}_{\alpha\omega}(B)$.

(viii) $\mathrm{int}_{\alpha\omega}(A) \cup \mathrm{int}_{\alpha\omega}(B) \subset \mathrm{int}_{\alpha\omega}(A \cup B)$.

**Proof.**

(iv) If $x \in A - D_{\alpha\omega}(X - A)$, then $x \notin D_{\alpha\omega}(X - A)$ and so there exists a $\alpha\omega$open set $U$ containing $x$ such that $U \cap (X - A) = \varphi$. Then $x \in U \subset A$ and hence $x \in \mathrm{int}_{\alpha\omega}(A)$, i.e., $A - D_{\alpha\omega}(X - A) \subset \mathrm{int}_{\alpha\omega}(A)$. On the other hand, if $x \in \mathrm{int}_{\alpha\omega}(A)$, then $x \notin D_{\alpha\omega}(X - A)$. Since $\mathrm{int}_{\alpha\omega}(A)$ is $\alpha\omega$- open and $\mathrm{int}_{\alpha\omega}(A) \cap (X - A) = \varphi$. Hence $\mathrm{int}_{\alpha\omega}(A) = A - D_{\alpha\omega}(X - A)$.

(v) $X - \mathrm{int}_{\alpha\omega}(A) = X - (A - D_{\alpha\omega}(X - A)) = (X - A) \cup D_{\alpha\omega}(X - A) =$

$\mathrm{cl}_{\alpha\omega}(X - A)$.

**Definition 3.7.** For a subset $A$ of a space $X$, $b_{\alpha\omega}(A) = A - \mathrm{int}_{\alpha\omega}(A)$ is said to be $\alpha\omega$-border of $A$.

**Theorem 3.8.** For a subset $A$ of a space $X$, the following statements hold:

(1) $b_{\alpha\omega}(A) \subset b(A)$, where $b(A)$ denotes the border of $A$.

(2) $A = \mathrm{int}_{\alpha\omega}(A) \cup b_{\alpha\omega}(A)$.

(3) $\mathrm{int}_{\alpha\omega}(A) \cap b_{\alpha\omega}(A) = \varphi$.

(4) $A$ is a $\alpha\omega$-open set if and only if $b_{\alpha\omega}(A) = \varphi$.

(5) $b_{\alpha\omega}(\mathrm{int}_{\alpha\omega}(A)) = \varphi$.

(6) $\mathrm{int}_{\alpha\omega}(b_{\alpha\omega}(A)) = \varphi$.

(7) $b_{\alpha\omega}(b_{\alpha\omega}(A)) = b_{\alpha\omega}(A)$.

(8) $b_{\alpha\omega}(A) = A \cap \mathrm{cl}_{\alpha\omega}(X - A)$.

(9) $b_{\alpha\omega}(A) = D_{\alpha\omega}(X - A)$.

**Proof.**

(6) If $x \in \mathrm{int}_{\alpha\omega}(b_{\alpha\omega}(A))$, then $x \in b_{\alpha\omega}(A)$. On the other hand, since $b_{\alpha\omega}(A) \subset A$, $x \in \mathrm{int}_{\alpha\omega}(b_{\alpha\omega}(A)) \subset \mathrm{int}_{\alpha\omega}(A)$. Hence $x \in \mathrm{int}_{\alpha\omega}(A) \cap b_{\alpha\omega}(A)$ which contradicts (3). Thus $\mathrm{int}_{\alpha\omega}(b_{\alpha\omega}(A)) = \varphi$.

(8) $b_{\alpha\omega}(A) = A - \mathrm{int}_{\alpha\omega}(A) = A - (X - \mathrm{cl}_{\alpha\omega}(X - A)) = A \cap \mathrm{cl}_{\alpha\omega}(X - A)$.

(9) $b_{\alpha\omega}(A) = A - \mathrm{int}_{\alpha\omega}(A) = A - (A - D_{\alpha\omega}(X - A)) = D_{\alpha\omega}(X - A)$.

**Definition 3.9.** For a subset $A$ of a space $X$, $Fr_{\alpha\omega}(A) = \mathrm{cl}_{\alpha\omega}(A) - \mathrm{int}_{\alpha\omega}(A)$ is said to be $\alpha\omega$-frontier of $A$.

**Theorem 3.10.** For a subset $A$ of a space $X$, the following statements hold:

(1) $Fr_{\alpha\omega}(A) \subset Fr(A)$, where $Fr(A)$ denotes the frontier of $A$.

(2) $\mathrm{cl}_{\alpha\omega}(A) = \mathrm{int}_{\alpha\omega}(A) \cup Fr_{\alpha\omega}(A)$

(3) $\mathrm{int}_{\alpha\omega}(A) \cap Fr_{\alpha\omega}(A) = \varphi$.

(4) $b_{\alpha\omega}(A) \subset Fr_{\alpha\omega}(A)$.

(5) $Fr_{\alpha\omega}(A) = b_{\alpha\omega}(A) \cup D_{\alpha\omega}(A)$.

(6) $A$ is a $\alpha\omega$-open set if and only if $Fr_{\alpha\omega}(A) = D_{\alpha\omega}(A)$.

(7) $Fr_{\alpha\omega}(A) = \mathrm{cl}_{\alpha\omega}(A) \cap \mathrm{cl}_{\alpha\omega}(X - A)$.

(8) $Fr_{\alpha\omega}(A) = Fr_{\alpha\omega}(X - A)$

(9) $Fr_{\alpha\omega}(A)$ is $\alpha\omega$- closed.

(10) $Fr_{\alpha\omega}(Fr_{\alpha\omega}(A)) \subset Fr_{\alpha\omega}(A)$.

(11) $Fr_{\alpha\omega}(\mathrm{int}_{\alpha\omega}(A)) \subset Fr_{\alpha\omega}(A)$.

(12) $Fr_{\alpha\omega}(\mathrm{cl}_{\alpha\omega}(A)) \subset Fr_{\alpha\omega}(A)$.

(13) $\mathrm{int}_{\alpha\omega}(A) = A - Fr_{\alpha\omega}(A)$.

**Proof.**

(2) $\mathrm{int}_{\alpha\omega}(A) \cup Fr_{\alpha\omega}(A) = \mathrm{int}_{\alpha\omega}(A) \cup (\mathrm{cl}_{\alpha\omega}(A) - \mathrm{int}_{\alpha\omega}(A)) = \mathrm{cl}_{\alpha\omega}(A)$.

(3) $\mathrm{int}_{\alpha\omega}(A) \cap Fr_{\alpha\omega}(A) = \mathrm{int}_{\alpha\omega}(A) \cap (\mathrm{cl}_{\alpha\omega}(A) - \mathrm{int}_{\alpha\omega}(A)) = \varphi$.

(5) Since $\mathrm{int}_{\alpha\omega}(A) \cup Fr_{\alpha\omega}(A) = \mathrm{int}_{\alpha\omega}(A) \cup b_{\alpha\omega}(A) \cup D_{\alpha\omega}(A)$, $Fr_{\alpha\omega}(A) = b_{\alpha\omega}(A) \cup D_{\alpha\omega}(A)$.

(7) $Fr_{\alpha\omega}(A) = \mathrm{cl}_{\alpha\omega}(A) - \mathrm{int}_{\alpha\omega}(A) = \mathrm{cl}_{\alpha\omega}(A) \cap \mathrm{cl}_{\alpha\omega}(X - A)$.

(9) $\mathrm{cl}_{\alpha\omega}(Fr_{\alpha\omega}(A)) = \mathrm{cl}_{\alpha\omega}(\mathrm{cl}_{\alpha\omega}(A) \cap \mathrm{cl}_{\alpha\omega}(X - A)) \subset \mathrm{cl}_{\alpha\omega}(\mathrm{cl}_{\alpha\omega}(A)) \cap \mathrm{cl}_{\alpha\omega}(\mathrm{cl}_{\alpha\omega}(X - A)) = Fr_{\alpha\omega}(A)$. Hence $Fr_{\alpha\omega}(A)$ is $\alpha\omega$- closed.

(10) $Fr_{\alpha\omega}(Fr_{\alpha\omega}(A)) = \mathrm{cl}_{\alpha\omega}(Fr_{\alpha\omega}(A)) \cap \mathrm{cl}_{\alpha\omega}(X - Fr_{\alpha\omega}(A)) \subset \mathrm{cl}_{\alpha\omega}(Fr_{\alpha\omega}(A)) = Fr_{\alpha\omega}(A)$.

(12) $Fr_{\alpha\omega}(\mathrm{cl}_{\alpha\omega}(A)) = \mathrm{cl}_{\alpha\omega}(\mathrm{cl}_{\alpha\omega}(A)) - \mathrm{int}_{\alpha\omega}(\mathrm{cl}_{\alpha\omega}(A)) = \mathrm{cl}_{\alpha\omega}(A) - \mathrm{int}_{\alpha\omega}(\mathrm{cl}_{\alpha\omega}(A)) = \mathrm{cl}_{\alpha\omega}(A) - \mathrm{int}_{\alpha\omega}(A) = Fr_{\alpha\omega}(A)$.

(13) $A - Fr_{\alpha\omega}(A) = A - (\mathrm{cl}_{\alpha\omega}(A) - \mathrm{int}_{\alpha\omega}(A)) = \mathrm{int}_{\alpha\omega}(A)$

In general the converse of (1) and (4) may not be true by the following Example.

**Example 3.11.** Let $X = \{a,b,c\}$ with topology $\tau = \{X,\varphi,\{a,b\}\}$. If $A = \{a\}$, then $Fr(A)$ does not contained in $Fr_{\alpha\omega}(A)$ and if $B = \{a,b\}$, then $Fr_{\alpha\omega}(B)$ does not contained in $b_{\alpha\omega}(B)$.

**Definition 3.12.** [6] A function $f : (X,\tau) \to (Y,\sigma)$ is $\alpha\omega$-continuous if $f^{-1}(V) \in \alpha\omega O(X)$ for every $V \in O(Y)$.

In the following theorem $J\alpha\omega.C.$ denote the set of points $x$ of $X$ for which a function $f : (X,\tau) \to (Y,\sigma)$ is not $\alpha\omega$-continuous.

**Theorem 3.13.** $J\alpha\omega.C.$ is identical with the union of the $\alpha\omega$-frontiers of the inverse images of $\alpha\omega$-open sets containing $f(x)$.

**Proof.** Suppose that $f$ is not $\alpha\omega$-continuous at a point $x$ of $X$. Then there exists an open set $V \subset Y$ containing $f(x)$ such that $f(U)$ is not a subset of $V$ for every $U \in \alpha\omega O(X)$ containing $x$. Hence we have $U \cap (X - f^{-1}(V)) 6= \varphi$ for every $U \in \alpha\omega O(X)$ containing $x$. It follows that $x \in \mathrm{cl}_{\alpha\omega}(X - f^{-1}(A))$. We also have $x \in f^{-1}(V) \subset \mathrm{cl}_{\alpha\omega}(f^{-1}(A))$. This means that $x \in Fr_{\alpha\omega}(f^{-1}(V))$. Now, let $f$ be $\alpha\omega$-continuous at $x \in X$ and $V \subset Y$ be any open set containing $f(x)$.
Then $x \in f^{-1}(V)$ is a $\alpha\omega$-open set of $X$. Thus $x \in \mathrm{int}_{\alpha\omega}(f^{-1}(V))$ and therefore $x \notin Fr_{\alpha\omega}(f^{-1}(V))$ for every open set $V$ containing $f(x)$.

**Definition 3.14.** For a subset $A$ of a space $X$, $Ext_{\alpha\omega}(A) = \mathrm{int}_{\alpha\omega}(X - A)$ is said to be $\alpha\omega$-exterior of $A$.

**Theorem 3.15.** For a subset $A$ of a space $X$, the following statements hold:

(1) $Ext(A) \subset Ext_{\alpha\omega}(A)$, where $Ext(A)$ denotes the exterior of $A$.

(2) $Ext_{\alpha\omega}(A)$ is $\alpha\omega$- open.

(3) $Ext_{\alpha\omega}(A) = \mathrm{int}_{\alpha\omega}(X - A) = X - \mathrm{cl}_{\alpha\omega}(A)$.

(4) $Ext_{\alpha\omega}(Ext_{\alpha\omega}(A)) = \mathrm{int}_{\alpha\omega}(\mathrm{cl}_{\alpha\omega}(A))$.

(5) If $A \subset B$, then $Ext_{\alpha\omega}(A) \supset Ext_{\alpha\omega}(B)$.

(6) $Ext_{\alpha\omega}(A \cup B) \subset Ext_{\alpha\omega}(A) \cup Ext_{\alpha\omega}(B)$.

(7) $Ext_{\alpha\omega}(X) = \varphi$.

(8) $Ext_{\alpha\omega}(\varphi) = X$.

(9) $Ext_{\alpha\omega}(A) = Ext_{\alpha\omega}(X - Ext_{\alpha\omega}(A))$.

(10) $\mathrm{int}_{\alpha\omega}(A) \subset Ext_{\alpha\omega}(Ext_{\alpha\omega}(A))$.

(11) $X = \mathrm{int}_{\alpha\omega}(A) \cup Ext_{\alpha\omega}(A) \cup Fr_{\alpha\omega}(A)$.

**Proof.**

(4) $Ext_{\alpha\omega}(Ext_{\alpha\omega}(A)) = Ext_{\alpha\omega}(X - \mathrm{cl}_{\alpha\omega}(A)) = \mathrm{int}_{\alpha\omega}(X - (X - \mathrm{cl}_{\alpha\omega}(A))) =$

$\mathrm{int}_{\alpha\omega}(\mathrm{cl}_{\alpha\omega}(A))$.

(9) $Ext_{\alpha\omega}(X - Ext_{\alpha\omega}(A)) = Ext_{\alpha\omega}(X - \mathrm{int}_{\alpha\omega}(X - A)) = \mathrm{int}_{\alpha\omega}(X - (X - \mathrm{int}_{\alpha\omega}(X - A))) = \mathrm{int}_{\alpha\omega}(\mathrm{int}_{\alpha\omega}(X - A)) = \mathrm{int}_{\alpha\omega}(X - A) = Ext_{\alpha\omega}(A)$.
(10) $\mathrm{int}_{\alpha\omega}(A) \subset \mathrm{int}_{\alpha\omega}(\mathrm{cl}_{\alpha\omega}(A)) = \mathrm{int}_{\alpha\omega}(X - \mathrm{int}_{\alpha\omega}(X - A)) = \mathrm{int}_{\alpha\omega}(X - Ext_{\alpha\omega}(A)) =$

$Ext_{\alpha\omega}(Ext_{\alpha\omega}(A))$.

In general the converse of (1) and (6) may not be true by the following Exam-

ple.

**Example 3.16.** Let $X = \{a,b,c\}$ with topology $\tau = \{X,\varphi,\{a,b\}\}$. If $A = \{a\}$ and $B = \{b\}$, then $Ext_{\alpha\omega}(A)$ does not contained in $Ext(A)$, $Ext_{\alpha\omega}(A \cup B)$ 6= $Ext_{\alpha\omega}(A) \cup Ext_{\alpha\omega}(B)$.

**Definition 3.17.** Let $X$ be a topological space. A set $A \subset X$ is said to be $\alpha\omega$-saturated if for every $x \in A$ it follows $\mathrm{cl}_{\alpha\omega}(\{x\}) \subset A$. The set of all $\alpha\omega$saturated sets in $X$ we denote by $B_{\alpha\omega}(X)$.

**Theorem 3.18.** Let $X$ be a topological space. Then $B_{\alpha\omega}(X)$ is a complete

Boolean set algebra.

**Proof.** We will prove that all the unions and complements of elements of $B_{\alpha\omega}(X)$ are members of $B_{\alpha\omega}(X)$. Obviously, only the proof regarding the complements is not trivial. Let $A \in B_{\alpha\omega}(X)$ and suppose that $\mathrm{cl}_{\alpha\omega}(\{x\})$ does not contained in $X - A$ for some $x \in X - A$. Then there exists $y \in A$ such that $y \in \mathrm{cl}_{\alpha\omega}(\{x\})$. It follows that $x,y$ have no disjoint neighbourhoods. Then $x \in \mathrm{cl}_{\alpha\omega}(\{y\})$. But this is a contradiction, because by the definition of $B_{\alpha\omega}(X)$ we have $\mathrm{cl}_{\alpha\omega}(\{y\}) \subset A$. Hence, $\mathrm{cl}_{\alpha\omega}(\{x\}) \subset X - A$ for every $x \in X - A$, which implies $X - A \in B_{\alpha\omega}(X)$.

**Corollary 3.19.** $B_{\alpha\omega}(X)$ contains every union and every intersection of $\alpha\omega$closed and $\alpha\omega$-open sets in $X$.

# 3 Grills: $\alpha\omega(\theta)$-convergence and $\alpha\omega(\theta)$- adherence

**Definition 4.1.** A grill G on a topological space $X$ is said to

(i) $\alpha\omega(\theta)$-adhere at $x \in X$ if for each $U \in \alpha\omega O(x)$ and each $G \in$G, $\mathrm{cl}_{\alpha\omega}(U) \cap G$ 6= $\varphi$,

(ii) $\alpha\omega(\theta)$-converge to a point $x \in X$ if for each $U \in \alpha\omega O(x)$, there is some $G \in$ G such that $G \subseteq \mathrm{cl}_{\alpha\omega}(U)$ (in this case we shall also say that G is

$\alpha\omega(\theta)$-convergent to $x$).

**Remark 4.2.** It at once follows that a grill G is $\alpha\omega(\theta)$-convergent to a point

n o $x \in X$ if and only if G contains the collection $\mathrm{cl}_{\alpha\omega}(U) : U \in \alpha\omega O(x)$ .

**Definition 4.3.** A filter F on a topological space $X$ is said to $\alpha\omega(\theta)$- adhere at $x \in X$ ($\alpha\omega(\theta)$-converge to $x \in X$) if for each $F \in$F and each $U \in \alpha\omega O(x)$ ,

$F \cap \mathrm{cl}_{\alpha\omega}(U)$ 6= $\varphi$ (resp. to each $U \in \alpha\omega O(x)$, there corresponds $F \in$F such that $F \subseteq \mathrm{cl}_{\alpha\omega}(U)$).

**Definition 4.4.** [10] If G is a grill (or a filter) on a space $X$, then the sec-

n tion of G, denoted by secG is given by secG = $A \subseteq X : A \cap G$ =6 $\varphi$, for all o
$G \in$G .

**Lemma 4.5.** [10]

(a) For any grill (filter) G on a space $X$, secG is a filter(resp. grill) on $X$.

(b) If F and G are respectively a filter and a grill on a space $X$ with F ⊆G, then there is an ultrafilter U on $X$ such that F⊆U⊆G.

**Theorem 4.6.** If a grill G on a topological space $X$, $\alpha\omega(\theta)$-adheres at some point $x \in X$, then G is $\alpha\omega(\theta)$-convergent to $x$.

**Proof.** Let a grill G on $X$, $\alpha\omega(\theta)$-adhere at $x \in X$. Then for each $U \in \alpha\omega O(x)$ and each $G \in$G, $cl_{\alpha\omega}(U) \cap G \neq \varphi$ so that $cl_{\alpha\omega}(U) \in$ secG, for each $U \in \alpha\omega O(x)$ and hence $X - cl_{\alpha\omega}(U) \notin$G . Then $cl_{\alpha\omega}(U) \in$G (as G is a grill and $X \in$G), for each $U \in \alpha\omega O(x)$. Hence G must $\alpha\omega(\theta)$-converge to $x$.

**Notation 4.7.** Let $X$ be a topological space. Then for any $x \in X$, we have

the following notation:

$$G(\alpha\omega(\theta),x) = \left\{ A \subseteq X : x \in \alpha\omega(\theta)\text{-cl}(A) \right\}$$

$$secG(\alpha\omega(\theta),x) = \left\{ A \subseteq X : A \cap G \neq \varphi, \text{ for all } G \in G(\alpha\omega(\theta),x) \right\}$$

In the next two theorems, we characterize the $\alpha\omega(\theta)$-adherence and $\alpha\omega(\theta)$- convergence of grills in terms of the above notations.

**Theorem 4.8.** A grill G on a space $X$, $\alpha\omega(\theta)$-adheres to a point $x \in X$ if and only if G⊆G$(\alpha\omega(\theta),x)$.

**Proof.** A grill G on a space $X$, $\alpha\omega(\theta)$-adheres at $x \in X$.

$\Rightarrow cl_{\alpha\omega}(U) \cap G \neq \varphi$, for all $U \in \alpha\omega O(x)$ and all $G \in$G

$$\Rightarrow x \in \alpha\omega(\theta)\text{-}cl(G), \text{ for all } G \in G$$

$$\Rightarrow G \in G(\alpha\omega(\theta),x), \text{ for all } G \in G$$

$$\Rightarrow G \subseteq G(\alpha\omega(\theta),x).$$

Conversely, let G⊆G$(\alpha\omega(\theta),x)$. Then for all $G \in$G, $x \in \alpha\omega(\theta)$-cl$(G)$, so that for all $U \in \alpha\omega O(x)$ and for all $G \in$G, $cl_{\alpha\omega}(U) \cap G \neq \varphi$. Hence G is $\alpha\omega(\theta)$-adheres at $x$.

**Theorem 4.9.** A grill G on a topological space $X$ is $\alpha\omega(\theta)$-convergent to a point $x$ of $X$ if and only if secG$(\alpha\omega(\theta),x)$ ⊆G.

**Proof.** Let G be a grill on $X$, $\alpha\omega(\theta)$-converging to $x \in X$. Then for each

$U \in \alpha\omega O(x)$ there exists $G \in$G such that $G \subseteq cl_{\alpha\omega}(U)$ and hence

$$cl_{\alpha\omega}(U) \in \mathcal{G} \text{ for each } U \in \alpha\omega O(x) \qquad (1)$$

Now, $B \in sec\mathcal{G}(\alpha\omega(\theta),x) \Rightarrow X - B \notin \mathcal{G}(\alpha\omega(\theta),x) \Rightarrow x \notin \alpha\omega(\theta)\text{-cl}(X - B) \Rightarrow$ there exists $U \in \alpha\omega O(x)$ such that $\text{cl}_{\alpha\omega}(U) \cap (X - B) = \varphi \Rightarrow \text{cl}_{\alpha\omega}(U) \subseteq B$, where

$U \in \alpha\omega O(x) \Rightarrow B \in \mathcal{G}$ (by (1)).
Conversely, let if possible, $\mathcal{G}$ not to $\alpha\omega(\theta)$-converge to $x$. Then for some $U \in \alpha\omega O(x)$, $\text{cl}_{\alpha\omega}(U) \notin \mathcal{G}$ and hence $\text{cl}_{\alpha\omega}(U) \notin sec\mathcal{G}(\alpha\omega(\theta),x)$. Thus for some $A \in \mathcal{G}(\alpha\omega(\theta),x)$ ,

$$A \cap cl_{\alpha\omega}(U) = \varphi \qquad (2)$$

But $A \in \mathcal{G}(\alpha\omega(\theta),x) \Rightarrow x \in \alpha\omega(\theta)\text{-cl}(A) \Rightarrow cl_{\alpha\omega}(U) \cap A \neq \varphi$, contradicting (2).

**Definition 4.10.** A non empty subset $A$ of a topological space $X$ is called $\alpha\omega$-closed relative to $X$ if for every cover $\mathcal{U}$ of $A$ by $\alpha\omega$-open sets of $X$, there exists a finite subset $\mathcal{U}_0$ of $\mathcal{U}$ such that $A \subseteq \cup\{\text{cl}_{\alpha\omega}(U) : U \in \mathcal{U}_0\}$. If, in addition, $A = X$, then $X$ is called a $\alpha\omega$-closed space.

**Theorem 4.11.** A subset $A$ of a topological space $X$ is $\alpha\omega$-closed relative to $X$ if and only if every grill $\mathcal{G}$ on $X$ with $A \in \mathcal{G}$, $\alpha\omega(\theta)$-converges to a point in $A$. **Proof.** Let $A$ be $\alpha\omega$-closed relative to $X$ and $\mathcal{G}$ a grill on $X$ satisfying $A \in \mathcal{G}$ such that $\mathcal{G}$ does not $\alpha\omega(\theta)$-converges to any $a \in A$. Then to each $a \in A$, there corresponds some $U_a \in \alpha\omega O(a)$ such that $\text{cl}_{\alpha\omega}(U_a) \notin \mathcal{G}$. Now $\{U_a : a \in A\}$ is a cover of $A$ by $\alpha\omega$-open sets of $X$. Then $A \subseteq \cup_{i=1}^{n} \text{cl}_{\alpha\omega}(U_{a_i}) = U$ (say), for some positive integer $n$. Since $\mathcal{G}$ is a grill, $U \notin \mathcal{G}$ and hence $A \notin \mathcal{G}$, which is a contradiction.

Conversely, let $A$ be not $\alpha\omega$-closed relative to $X$. Then for some cover $\mathcal{U} = \{U_\alpha : \alpha \in \Lambda\}$ of $A$ by $\alpha\omega$-open sets of $X$, $\mathcal{F} = \{A - \cup_{\alpha\in\Lambda_0}\text{cl}_{\alpha\omega}(U_\alpha) : \Lambda_0$ is a finite subset of $\Lambda\}$ is a filterbase on $X$. Then the family $\mathcal{F}$ can be extended to an ultrafilter $\mathcal{F}^*$ on $X$. Then $\mathcal{F}^*$ is a grill on $X$ with $A \in \mathcal{F}^*$. Now for each $x \in A$, there must exist $\beta \in \Lambda$ such that $x \in U_\beta$, as $\mathcal{U}$ is a cover of $A$. Then for any $G \in \mathcal{F}^*$, $G \cap (A - \text{cl}_{\alpha\omega}(U_\beta)) \neq \varphi$, so that $G$ does not contained in $\text{cl}_{\alpha\omega}(U_\beta)$, for all $G \in \mathcal{G}$. Hence $\mathcal{F}^*$ cannot $\alpha\omega(\theta)$-converge to any point of $A$. The contradiction proves the desired result.

## References

[1] Choquent, Sur les notions de filtre et grille, *Comptes Rendus Acad. Sci. Paris*, 224, (1947), 171-173.

[2] N. Levine, semi-open sets and semi-continuity in topological spaces, *Amer. Math. Monthly*, 70(1963), 36-41.

[3] A.S. Mashhour, M.E. Abd El-Monsef and S.N. El-Deeb, On pre continuous and weak pre continuous mappings, *Proc. Math. Phys. Soc.*, Egypt, 53 (1982), 47-53.

[4] M. N. Mukherjee and B. Roy, *p*-closed topological spaces in terms of grills, *Hacettepe Journal of Mathematics and Statistics*, Vol. 35(2), (2006), 147154.

[5] O. Njastad, On some classes of nearly open sets, *Pacific J. Math.*, 15(1965) , 961- 970.

[6] M. Parimala , R. Udhayakumar , R. Jeevitha , V. Biju, On $\alpha\omega$-closed sets in topological spaces, *International Journal of Pure and Applied Mathematics*, 115(5), 2017, 1049-1056.

[7] M. Parimala, M. Karthika, F. Smarandache and S. Broumi, On $\alpha\omega$- closed sets and its connectedness in terms of neutrosophic topological spaces, *International Journal of neutrosophic Science (IJNS)*,2(2)(2020), 82-88.

[8] M. Parimala, C. Ozel, R Udhayakumar, On Ultra Separation Axioms via $\alpha\omega$-Open sets, *Advances in Algebra and Analysis*, (2018) 97-102. Springer (book series)

[9] P. Sundaram and M. Shrik John, On $\cap g$-closed sets in topology, *Acta Ciencia Indica*, 4(2000),389-392.

[10] W. J. Thron, Proximity Structure and grills, *Math. Ann.*, (1973), 35-62.

[11] M.K.R.S. Veera Kumar, $\cap g$-locally closed sets and $\cap GLC$-functions, *Indian J. Math*., 43(2)(2001), 231-247

[12] N.V. Velicko, *H*-closed topological spaces, *Amer. Math. Soc. Transl.*,

78(1968), 103-118.

[13] S. Willard, General Topology, *Addison - Wesley,* Reading, Mass, USA

(1970).