International Journal of Modern Agriculture, Volume 10, No.2, 2021
ISSN: 4405 - 4411

Stability of the Compressed Rod by the Scheme of Two-Module Material beyond the Limit of Elasticity

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Abstract. It is shown that at an infinitesimal bending of a compressed rod beyond the elastic limit, the secant modulus of its longitudinal fibers both in the zones of additional loading and unloading moves along an infinitely small section of the tangent to the critical point on the compression diagram $\sigma_i - \varepsilon_i$. Each of the longitudinal fibers of a compressed rod under conditions of infinitesimal bending has its own secant modulus, which linearly depends on the vertical coordinate *z*; in the zone of additional loading it decreases, and in the zone of unloading it increases.

Keywords. Stability; compression diagram; secant modulus; tangent modulus; rigidity; static moment.

Introduction. The phenomenon of instability is inherent not only in compressed rods, but also in thin plates, shells and, in general, various thin-walled structures made of rods, plates and shells, widely used in modern technology.

Loss of stability leads tostructural failure. Therefore, determining the magnitude of the critical load is an important practical task in design. Its relevance is especially substantial for the elements of building structures and aircraft, where it is vitally necessary to reach the maximum possible weight reduction, for the creation of light optimal forms of structures, taking into account the strength, rigidity, stability.

For the first time, the problem of the stability of a rod beyond the proportionality limit was considered in 1889. Engesser [1] proposed that at the moment of loss of stability, additional loading of some fibers and unloading of other fibers occur with a single tangent modulus. The load calculated under this assumption is called tangentially modular load. Later, Engesser and Karman solved the same problem, based on a different assumption, assuming that the unloading occurs according to a linear law, and introduced the so-called reduced modulus into consideration.The load calculated in this wayis called modular. Engesser's earlier work was recognized as erroneous. Nevertheless, the discrepancy between theory and experiment continued to persist against the new theory, since the experimental results systematically gave lower values of the critical load compared to the reduced modular one, being in better agreement with the tangentially modular load.

The theory of stability beyond the elastic limit was further developed in the studies conducted byShanley [2], with a carefully stated experiment; he established that the curvature of a compressed rod begins under tangential-modular load. Taking this statement as a postulate, the author made a theoretical analysis of the supercritical behavior of the rod at large displacements and showed that the given modular load is an asymptote, achieved at an infinitely large deflection value. Since the above analysis contains both concepts of critical load(relatively modular and reduced modular loads), the issue of the law of material unloading at the time of instability and after the release of this work remained open. The following studies should be indicated except the listed ones [4-15], the authors provided solutions to many problems related to the stability of structural elements.

Discussion. We assume that unloading occurs along a straight line $M_0 - 2$ parallel to the tangent $M_0 - 2$ that refer to the initial point of the diagram $\sigma - \varepsilon$ at an infinitesimal bending of the rod (at the time of bifurcation); therefore, an instantaneous break of tangent $I - I$ occurs (Fig. 1). In this formulation of the problem the stability of the rod, its material is two-module; in the additional loading zone, its module is tangent E_k, and in the unloading zone, we denote it by E, for $E > E_k$.

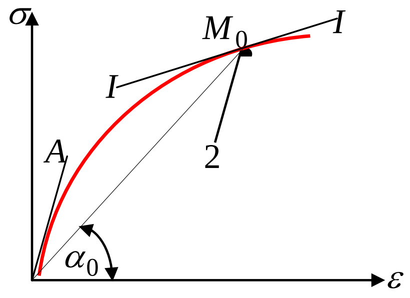

Fig. 1. Compression diagram of a two-module material of the rod.

The secant modulus ψ_1 in the additional loading zone is determined by the formula [3]:

$$\psi = \psi_0 \left[1 + \frac{\Delta\chi}{\varepsilon_0} z \left(1 - \frac{E_k}{\psi_0} \right) \right], \tag{1}$$

and the secant modulus ψ_2, related to the unloading zone is determined by the same formula, if modulus E_k is replaced by E:

$$\psi_2 = \psi_0 \left[1 - \frac{\Delta\chi}{\varepsilon_0} z \left(1 - \frac{E}{\psi_0} \right) \right]. \tag{2}$$

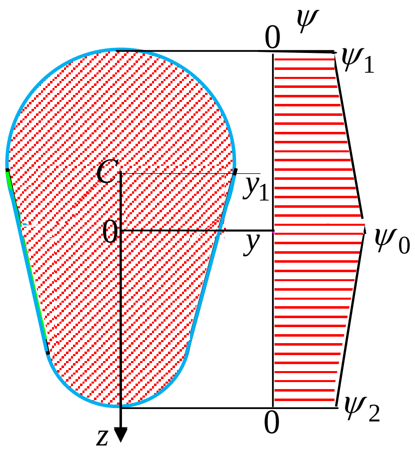

Fig. 2. Diagram of changes in secant modulus.

Since the modulus E is greater than the secant modulus of the critical point M_0, equal to $\psi_0 = tg\alpha_0$ (Fig. 1), the secant modulus ψ_2 isless than ψ_0 and, therefore, for a two-module material, the secant modulus in the unloading zone also decreases, as in the additional loading zone (Fig. 2); with a smooth transition of the straight line $M_0 - 1$ to the position $M_0 - 2$, as shown in [1], the secant modulus ψ_2 is greater than ψ_0 in the unloading zone.

It should be noted that in formulas (1) and (2), by which the moduli ψ_1 and ψ_2 are determined, the infinitesimal bending strain $\Delta\chi = -\dfrac{d^2\Delta w}{dx^2}$ is a positive value.

For further calculations, it is advisable to represent the formula (2) in the form:

$$\psi_2 = \psi_0\left[1 + \frac{\Delta\chi}{\varepsilon} z\left(1 - \frac{E_k}{\psi_0}\right)\right] - \frac{\Delta\chi}{\varepsilon} zE^*. \qquad (3)$$

where $E^* = E - E_k$,

We write down the formulas for rigidities $I_1;\ I_2; I_3$. Since the neutral axis in the considered case of a two-module material does not coincide with the central axis y_1, and the secant moduli ψ_1 and ψ_2 are determined by different dependencies (Fig. 2), the expressions for the rigidityshould be written separately by zones:

$$I_1 = \int_A \psi dA = \int_{A_1} \psi_1 dA + \int_{A_2} \psi_2 dA =$$

$$= \psi_0 \int_{A_1} \left[1 + \frac{\Delta\chi}{\varepsilon_0} z\left(1 - \frac{E_k}{\psi_0}\right)\right] dA +$$

$$+ \psi_0 \int_{A_2} \left[1 + \frac{\Delta\chi}{\varepsilon_0} z\left(1 - \frac{E_k}{\psi_0}\right)\right] dA - \frac{\Delta\chi}{\varepsilon_0} E^* \int_{A_2} zdA.$$

or

$$I_1 = \psi_0 \int_A \left[1 + \frac{\Delta\chi}{\varepsilon_0} z\left(1 - \frac{E_k}{\psi_0}\right)\right] dA - \frac{\Delta\chi}{\varepsilon_0} E^* \int_{A_2} zdA =$$

$$= \psi_0 A + \frac{\Delta\chi}{\varepsilon_0}(\psi_0 - E_k)\int_A zdA - \frac{\Delta\chi}{\varepsilon_0} E^*.$$

The first integral on the right-hand side of this expression represents the static moment S of the cross-section relative to the neutral axis that does not coincide with the central axis; it is non-zero.

The second integral on the right side represents the static moment of the lower (second) part of the section (Fig. 2) relative to the neutral axis; let us denote it by S_2:

$$S_2 = \int_{A_2} zdA. \qquad (4)$$

The formula for rigidity I_1 takes the form:

$$I_1 = \psi_0 A + \frac{\Delta\chi}{\varepsilon_0}(\psi_0 - E_k)S - \frac{\Delta\chi}{\varepsilon_0} E^* S_2. \quad (5)$$

Theexpressionforrigidity I_2 is written as

$$I_2 = \int_A \psi z dA = \psi_0 \int_{A_1} \left[1 + \frac{\Delta\chi}{\varepsilon_0} z \left(1 - \frac{E_k}{\psi_0} \right) \right] z dA +$$

$$+ \psi_0 \int_{A_2} \left[1 + \frac{\Delta\chi z}{\varepsilon_0} \left(1 - \frac{E_k}{\psi_0} \right) \right] z dA -$$

$$- \frac{\Delta\chi E^*}{\varepsilon_0} \int_{A_2} z^2 dA,$$

or

$$I_2 = \psi_0 \int_A \left[1 + \frac{\Delta\chi}{\varepsilon_0} z \left(1 - \frac{E_k}{\psi_0} \right) \right] z dA -$$

$$- \frac{\Delta\chi}{\varepsilon_0} E^* \int_{A_2} z^2 dA = = \int_A z dA +$$

$$+ \frac{\Delta\chi}{\varepsilon_0}(\psi_0 - E_k) \int_A z^2 dA - \frac{\Delta\chi}{\varepsilon_0} E^* \int_{A_2} z^2 dA.$$

The first integral on the right-hand side, as indicated above, is the static moment S of the cross-section relative to the neutral axis; the second integral is the moment of inertia I_y of the cross section relative to the neutral axis; the third integral is the moment of inertia of the lower (second) part of the cross section; we denote it by B_2:

$$B_2 = \int_{A_2} z^2 dA. \quad (6)$$

The formula for rigidity I_2 takes the form:

$$I_2 = \psi_0 S + \frac{\Delta\chi}{\varepsilon_0}(\psi_0 - E_k) I_y - \frac{\Delta\chi}{\varepsilon_0} E^* B_2. \quad (7)$$

Theexpressionforrigidity I_3 is written as

$$I_3 = \int_A \psi z^2 dA = \psi_0 \int_{A_1} \left[1 + \frac{\Delta\chi}{\varepsilon_0} z \left(1 - \frac{E_k}{\psi_0} \right) \right] z^2 dA +$$

$$+ \psi_0 \int_{A_2} \left[1 + \frac{\Delta\chi z}{\varepsilon_0} \left(1 - \frac{E_k}{\psi_0} \right) \right] z^2 dA - \frac{\Delta\chi E^*}{\varepsilon_0} \int_{A_2} z^3 dA =$$

$$\psi_0 \int_A z^2 dA + \frac{\Delta\chi}{\varepsilon_0} (\psi_0 - E_k) \int_A z^3 dA - \frac{\Delta\chi E^*}{\varepsilon_0} \int_{A_2} z^3 dA.$$

The first integral on the right-hand side is the moment of inertia I_y of the cross section relative to the neutral axis; the second and third integrals represent new geometric characteristics of the cross section, which relative to the coordinate z are of a higher order than the moment of inertia of the plane section. These new geometric characteristics of the section we denote by C:

$$C = \int_A z^3 dA. \; C_2 = \int_{A_2} z^3 dA. \tag{8}$$

The formula for rigidity I_3 takes the form:

$$I_3 = \psi_0 I_y + \frac{\Delta\chi}{\varepsilon_0} (\psi_0 - E_k) C - \frac{\Delta\chi}{\varepsilon_0} E^* C_2. \tag{9}$$

We expand $\Delta M = -\Delta\chi I_3 + \varepsilon_0 I_2$ the basic equation for determining the infinitesimal internal bending moment:

$$\Delta M = -\Delta\chi I_3 + \varepsilon_0 I_2 = -\Delta\chi \Big[\psi_0 I_y +$$

$$+ \frac{\Delta\chi}{\varepsilon_0} (\psi_0 - E_k) C - \frac{\Delta\chi}{\varepsilon_0} E_k C \Big] +$$

$$+ \varepsilon_0 \left[\psi_0 S + \frac{\Delta\chi}{\varepsilon_0} (\psi_0 - E_k) I_y - \frac{\Delta\chi}{\varepsilon_0} E^* B_2 \right],$$

or

$$\Delta M = -\Delta\chi E_k I_y + \varepsilon_0 \psi_0 S -$$

$$- \frac{(\Delta\chi)^2}{\varepsilon_0} \left[(\psi_0 - E_k) C - E^* C_2 \right] - \Delta\chi E^* B_2. \tag{10}$$

Since the static moment S, taken relative to the neutral axis, is a finite value, then the middle term in the right-hand side of formula (10), equal to $\varepsilon_0 \psi_0 S$, is also a finite value. Then equation (10) is impossible, because the remaining terms of this equation are infinitely small quantities. Therefore, it is necessary to consider the static moment S as infinitely small quantity, that is, to assume that the neutral axis y merges with the central axis of the section y_1 (Fig. 2).

Consequently, the point M_0 on the compression diagram (Fig. 1) is not special and the tangent $I-I$ goes into position M_0-2 smoothly without breaking.

With an infinitely small bending of the rod, the tangent line $I-I$ remains common both for the additional loading zone I and for the unloading zone II (Fig. 2). This is only possible if we assume that its material is one-modular.

The same conclusion can be reached if we consider the expression $N=\varepsilon_0 I_1-\Delta\chi I_2$ for the longitudinal force N under conditions of a two-module material.

Substituting expressions for rigidities (5) and (7) into longitudinal force formulas, we have:

$$N=\varepsilon_0 I_1-\Delta\chi I_2=\varepsilon_0\left[\psi_0 A+\right.$$
$$\left.+\frac{\Delta\chi}{\varepsilon_0}(\psi_0-E_k)S-\frac{\Delta\chi}{\varepsilon_0}E^*S_2\right]-$$
$$-\Delta\chi\left[\psi_0 S+\frac{\Delta\chi}{\varepsilon_0}(\psi_0-E_k)I_y-\frac{\Delta\chi}{\varepsilon_0}E_k B_2\right],$$

or

$$N=\varepsilon_0\psi_0 A-\frac{\Delta\chi}{\varepsilon_0}\left(E_k S+E^*S_2\right)-$$
$$-\frac{\Delta\chi^2}{\varepsilon_0}\left[(\psi_0-E_k)I_y-E^*B_2\right] \qquad (11)$$

The first term on the right-hand side $\varepsilon_0\psi_0 A=\sigma_0 A$ is equal to the external compressive force F and is balanced by it, the last term can be eliminated since it is an infinitely small quantity of a higher order than the other terms in Eq. (11); the middle term $\frac{\Delta\chi}{\varepsilon_0}\left(E_k S+E^*S_2\right)$ from the equilibrium condition must be equal to zero.

This is possible if:

$$S=0;\quad E^*=E-E_k=0. \qquad (12)$$

Conclusion

The first condition requires that the neutral axis of the cross-section y coincides with the central axis y_1 (Fig. 2); the second condition will be fulfilled if at the initial stage of the bifurcation of equilibrium states, at an infinitesimal bending of the rod, under unloading, there will be one modulus E_k in the cross section.

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