

S-Strong Anti Fuzzy Graph and Its Applications In Agriculture

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Abstract:

In this paper, the concept of operation on anti fuzzy graphs with the property of S-strong is introduced. The operations on anti fuzzy graph such as anti cartesian product, anti join, anti union and anti composition is discussed. An operation on same types of anti fuzzy graphs is performed and the S-strong property on the resulting anti fuzzy graph is discussed. The applications of S-Strong concept in the inheritance of pea plants, snapdragon flowers and white bulls with black cows are discussed.

Keywords: *Anti fuzzy graph, S-strong, Anti cartesian product, Anti fuzzy tree.*

Mathematical Classification: 05C62, 05C69, 05C72, 05C76, 05E99.

I. Introduction

In 2012, Mohaamad Akram [1] proposed the anti fuzzy structure concept on graph and the connectedness on anti fuzzy graph. In 2016, R. Seethalakshmi and R. B. Gnanajothi [8] introduced the definition of anti fuzzy graph on operations of anti fuzzy graphs such as union and join on anti fuzzy graphs. In the same period, R. Muthuraj and A. Sasireka [7] introduced the concept of anti fuzzy graph and the concept some operations such as anti union, anti join, anti cartesian product and anti composition on anti fuzzy graphs are defined. In this paper, the concept of S-strong property with operations on anti fuzzy graphs such as anti union, anti join, anti cartesian product and anti composition are introduced. Some theorems and results are derived on them.

II. Preliminaries

Let us discuss the basic concepts of anti fuzzy graph in this section. Notations and more formal definitions are followed as in [6 - 8].

Definition 2.1 [7]

An anti fuzzy graph $G_A = (\sigma, \mu)$ is a pair of functions $\sigma : V \rightarrow [0,1]$ and $\mu : V \times V \rightarrow [0,1]$, with $\mu(u,v) \geq \sigma(u) \vee \sigma(v)$ for all $u, v \in V$.

Note

μ is considered as reflexive and symmetric. In all examples σ is chosen suitably. i.e., undirected anti fuzzy graphs are only considered.

Notation:

Without loss of generality let us simply use the letter G_A to denote an anti fuzzy graph.

Definition 2.2 [6]

Let $G_{A_1}(\sigma_1, \mu_1)$ and $G_{A_2}(\sigma_2, \mu_2)$ be two anti fuzzy graphs with $G_{A_1}^*$ and $G_{A_2}^*$. Let $G_A^* = G_{A_1}^* \bar{\cup} G_{A_2}^* = (V(G_{A_1}) \bar{\cup} V(G_{A_2}), E(G_{A_1}) \bar{\cup} E(G_{A_2}))$ be an anti union of anti fuzzy graphs of $G_{A_1}^*$ and $G_{A_2}^*$. The anti union of two anti fuzzy graphs G_{A_1} and G_{A_2} is an anti fuzzy graph $G_A = G_{A_1} \bar{\cup} G_{A_2} = (\sigma_1 \bar{\cup} \sigma_2, \mu_1 \bar{\cup} \mu_2)$ is defined by

$$\text{If } V_1 \cap V_2 = \emptyset \text{ then } (\sigma_1 \bar{\cup} \sigma_2)(u) = \begin{cases} \sigma_1(u) & \text{if } u \in V(G_{A_1}) \setminus V(G_{A_2}) \\ \sigma_2(u) & \text{if } u \in V(G_{A_2}) \setminus V(G_{A_1}) \end{cases}$$

$$(\mu_1 \bar{\cup} \mu_2)(u,v) = \begin{cases} \mu_1(u,v) & \text{if } uv \in E(G_{A_1}) \setminus E(G_{A_2}) \\ \mu_2(u,v) & \text{if } uv \in E(G_{A_2}) \setminus E(G_{A_1}) \end{cases}$$

and If $V_1 \cap V_2 \neq \emptyset$ then $(\sigma_1 \bar{\cup} \sigma_2)(u) = \sigma_1(u) \vee \sigma_2(u)$ if $u \in V(G_{A_1}) \cap V(G_{A_2})$.
 $(\mu_1 \bar{\cup} \mu_2)(u,v) = \mu_1(u,v) \vee \mu_2(u,v)$ if $(u,v) \in E(G_{A_1}) \cap E(G_{A_2})$.

Definition 2.3 [6]

Consider an anti join $G_A^* = G_{A_1}^* \bar{\cap} G_{A_2}^* = (V(G_{A_1}) \cup V(G_{A_2}), E(G_{A_1}) \cup E(G_{A_2}) \cup E')$ of anti fuzzy graphs where E' is the set of all edges joining the vertices of $V(G_{A_1})$ and $V(G_{A_2})$. Here assume that $V(G_{A_1}) \cap V(G_{A_2}) = \emptyset$. Then the anti join of anti fuzzy graphs G_{A_1} and G_{A_2} is an anti fuzzy graph $G_A = G_{A_1} \bar{\cap} G_{A_2} = (\sigma_1 \bar{\cap} \sigma_2, \mu_1 \bar{\cap} \mu_2)$ is defined by

$$(\sigma_1 \bar{\cap} \sigma_2)(u) = (\sigma_1 \cup \sigma_2)(u) \text{ if } u \in V(G_{A_1}) \cup V(G_{A_2})$$

$$(\mu_1 \bar{\cap} \mu_2)(u,v) = (\mu_1 \cup \mu_2)(u,v) \text{ if } (u,v) \in E(G_{A_1}) \cup E(G_{A_2}) \text{ and}$$

$$(\mu_1 \bar{\cap} \mu_2)(u,v) = \max\{(\sigma_1(u), \sigma_2(v))\} \text{ if } (u,v) \in E'.$$

Definition 2.4 [6]

Let $G_A^* = G_{A_1}^* \bar{\times} G_{A_2}^* = (V, E')$ be the anti cartesian product of anti fuzzy graphs where $V = V(G_{A_1}) \bar{\times} V(G_{A_2})$ and $E' = \{((u_1, u_2), (u_1, v_2)) / u_1 \in V(G_{A_1}), (u_2, v_2) \in E(G_{A_2})\} \cup \{(u_1, w_2), (v_1, w_2) / w_2 \in V(G_{A_2}), (u_1, v_1) \in E(G_{A_1})\}$. Then the anti cartesian product of two anti fuzzy graphs, $G_A = G_{A_1} \bar{\times} G_{A_2} = (\sigma_1 \bar{\times} \sigma_2, \mu_1 \bar{\times} \mu_2)$ is an anti fuzzy graph and is defined by

$(\sigma_1 \bar{\times} \sigma_2)(u_1, u_2) = \max \{ \sigma_1(u_1), \sigma_2(u_2) \}$ for all $(u_1, u_2) \in V$,

$(\mu_1 \bar{\times} \mu_2)((u_1, u_2), (u_1, v_2)) = \max \{ \sigma_1(u_1), \mu_2(u_2, v_2) \}$ for all $u_1 \in V(G_{A_1})$ and $(u_2, v_2) \in E(G_{A_2})$,

$(\mu_1 \bar{\times} \mu_2)((u_1, w_2), (v_1, w_2)) = \max \{ \sigma_2(w_2), \mu_1(u_1, v_1) \}$ for all $w_2 \in V(G_{A_2})$ and $(u_1, v_1) \in E(G_{A_1})$,

Then the anti fuzzy graph $G_A = (\sigma_1 \bar{\times} \sigma_2, \mu_1 \bar{\times} \mu_2)$ is said to be the anti cartesian product of $G_{A_1} = (\sigma_1, \mu_1)$ and $G_{A_2} = (\sigma_2, \mu_2)$.

Definition 2.5 [6]

Let $G_A^* = G_{A_1}^* \circ G_{A_2}^* = (V, E)$ be an anti composition of anti fuzzy graphs where $V = V(G_{A_1}) \bar{\times} V(G_{A_2})$ and $E = \{((u_1, u_2), (u_1, v_2)) / u_1 \in V(G_{A_1}), (u_2, v_2) \in E(G_{A_2})\} \cup \{(u_1, w_2), (v_1, w_2) / w_2 \in V(G_{A_2}), (u_1, v_1) \in E(G_{A_1})\} \cup \{(u_1, u_2), (v_1, v_2) / (u_1, v_1) \in E(G_{A_1}), u_2 \neq v_2\}$. Then the anti composition of anti fuzzy graphs, $G_A = G_{A_1} \bar{\circ} G_{A_2} = (\sigma_1 \bar{\circ} \sigma_2, \mu_1 \bar{\circ} \mu_2)$ is an anti fuzzy graph and is defined by

$(\sigma_1 \bar{\circ} \sigma_2)(u_1, u_2) = \max \{ \sigma_1(u_1), \sigma_2(u_2) \}$ for all $(u_1, u_2) \in V(G_A)$

$(\mu_1 \bar{\circ} \mu_2)((u_1, u_2), (u_1, v_2)) = \max \{ \sigma_1(u_1), \mu_2(u_2, v_2) \}$ for all $u_1 \in V(G_{A_1})$ and $(u_2, v_2) \in E(G_{A_2})$,

$(\mu_1 \bar{\circ} \mu_2)((u_1, w_2), (v_1, w_2)) = \max \{ \sigma_2(w_2), \mu_1(u_1, v_1) \}$ for all $w_2 \in V(G_{A_2})$ and $(u_1, v_1) \in E(G_{A_1})$,

$(\mu_1 \bar{\circ} \mu_2)((u_1, u_2), (v_1, v_2)) = \max \{ \sigma_2(u_2), \sigma(v_2), \mu_1(u_1, v_1) \}$ for all $(u_1, u_2), (v_1, v_2) \in E - E''$.

Where $E'' = \{((u_1, u_2), (v_1, v_2)) / u_1 \in V(G_{A_1}), (u_2, v_2) \in E(G_{A_2})\} \cup \{(u_1, w_2), (v_1, w_2) / w_2 \in V(G_{A_2}), (u_1, v_1) \in E(G_{A_1})\}$.

III. Operations on S-strong Anti Fuzzy Graph

The cartesian product and disjoint sum of graphs play a important role and have various fascinating algebraic properties. In such case, we consider the operations on fuzzy graphs under which S-strong property is conserved.

Definition 3.1

The subset S is said to be a S-strong anti fuzzy subgraph if $\mu(u, v) = \sigma(u) \vee \sigma(v)$ for all $(u, v) \in E(G_A)$ then the subset S is complete anti fuzzy graph.

Definition 3.2

Let (σ, μ) be an anti fuzzy subgraph of $G_A(V, E)$. Denote by E^* the set of all $(u, v) \in E$ for which the M-strong property fails. That is, $(u, v) \in E^*$. if and only if $\mu(u, v) > \sigma(u) \vee \sigma(v)$.

Theorem 3.3

If G_{A_1} and G_{A_2} are S-strong anti fuzzy graphs then $G_{A_1} \bar{\times} G_{A_2}$, $G_{A_1} \bar{\cup} G_{A_2}$ and $G_{A_1} \bar{\circ} G_{A_2}$ are also S-strong.

Proof

Let us consider that G_{A_1} and G_{A_2} are S-strong anti fuzzy graphs.

Then by the definition of anti cartesian product of anti fuzzy graph,

$$\begin{aligned} (\mu_1 \bar{\times} \mu_2)((u_1, u_2), (u_1, v_2)) &= \sigma_1(u_1) \vee \mu_2(u_2, v_2) \} \text{ for all } u_1 \in V(G_{A_1}) \text{ and } (u_2, v_2) \in E(G_{A_2}) \\ &= \sigma_1(u_1) \vee \sigma_2(u_2) \vee \sigma_2(v_2) \\ &= (\sigma_1 \bar{\times} \sigma_2)(u_1, u_2) \vee (\sigma_1 \bar{\times} \sigma_2)(u_1, v_2) \end{aligned}$$

Therefore, $(\mu_1 \bar{\times} \mu_2)((u_1, u_2), (u_1, v_2)) = (\sigma_1 \bar{\times} \sigma_2)(u_1, u_2) \vee (\sigma_1 \bar{\times} \sigma_2)(u_1, v_2)$

Consider, $(\mu_1 \bar{\times} \mu_2)((u_1, w_2), (v_1, w_2)) = \sigma_2(w_2) \vee \mu_1(u_1, v_1)$ for all $w_2 \in V_2$ and $(u_1, v_1) \in E(G_{A_1})$,

$$\begin{aligned} &= \sigma_2(w_2) \vee \sigma_1(u_1) \vee \sigma_1(v_1) \\ &= (\sigma_1 \bar{\times} \sigma_2)(u_1, w_2) \vee (\sigma_1 \bar{\times} \sigma_2)(v_1, w_2) \end{aligned}$$

Therefore, $(\mu_1 \bar{\times} \mu_2)((u_1, w_2), (v_1, w_2)) = (\sigma_1 \bar{\times} \sigma_2)(u_1, w_2) \vee (\sigma_1 \bar{\times} \sigma_2)(v_1, w_2)$.

$(\sigma_1 \bar{\times} \sigma_2, \mu_1 \bar{\times} \mu_2)$ is an S-strong anti fuzzy graph.

By the definition of anti join of anti fuzzy graph,

$$\begin{aligned} (\sigma_1 \bar{\cup} \sigma_2)(u) &= (\sigma_1 \cup \sigma_2)(u) \text{ if } u \in V(G_{A_1}) \cup V(G_{A_2}) \\ (\mu_1 \bar{\cup} \mu_2)(u, v) &= (\mu_1 \cup \mu_2)(u, v) \text{ if } (u, v) \in E(G_{A_1}) \cup E(G_{A_2}) \text{ and} \\ (\mu_1 \bar{\cup} \mu_2)(u, v) &= (\sigma_1(u) \vee \sigma_2(v)) \text{ if } (u, v) \in E'. \end{aligned}$$

Therefore, $G_{A_1} \bar{\cup} G_{A_2}$ is an S-strong anti fuzzy graph.

By the definition of anti composition of anti fuzzy graph,

$$\begin{aligned} (\mu_1 \bar{\circ} \mu_2)((u_1, u_2), (v_1, v_2)) &= \sigma_2(u_2) \vee \sigma_2(v_2) \vee \mu_1(u_1, v_1) \text{ for all } (u_1, u_2)(v_1, v_2) \in E - E'' \\ &= \sigma_2(u_2) \vee \sigma_2(v_2) \vee \sigma_1(u_1) \vee \sigma_1(v_1) \\ &= (\sigma_1 \bar{\times} \sigma_2)(u_1, u_2) \vee (\sigma_1 \bar{\times} \sigma_2)(v_1, v_2) \end{aligned}$$

Therefore, $G_{A_1} \bar{\circ} G_{A_2}$ is an S-strong anti fuzzy graph.

Proposition 3.4

If G_{A_1} and G_{A_2} are two S-strong anti fuzzy graphs then the anti union of S-strong anti fuzzy graphs need not be S-strong .

Theorem 3.5

If $G_{A_1} \bar{\times} G_{A_2}$ is a S-strong anti fuzzy graph then at least one of G_{A_1} or G_{A_2} must be S-strong.

Proof:

Suppose assume that G_{A_1} and G_{A_2} are not S-strong anti fuzzy graphs. Then any one of the edge $(u_1, v_1) \in E(G_{A_1})$ and any one of the edge $(u_2, v_2) \in E(G_{A_2})$ such that

$$\mu_1(u_1, v_1) > \sigma_1(u_1) \vee \sigma_1(v_1) \text{ and } \mu_2(u_2, v_2) > \sigma_2(u_2) \vee \sigma_2(v_2) \longrightarrow (i)$$

Without loss of generality, assume that

$$\mu_2(u_2, v_2) \geq \mu_1(u_1, v_1) \geq \sigma_1(u_1) \vee \sigma_1(v_1) \geq \sigma_2(u_2) \vee \sigma_2(v_2) \longrightarrow (ii)$$

Now consider $((u_1, u_2), (u_1, v_2)) \in E$ where E is defined in the definition of anti cartesian product of anti fuzzy graphs. Also by this definition the equality (i)

$$\begin{aligned} (\mu_1 \bar{\times} \mu_2)((u_1, u_2), (u_1, v_2)) &= \sigma_1(u_1) \vee \mu_2(u_2, v_2) \\ &\leq \sigma_1(u_1) \vee \sigma_2(u_2) \vee \sigma_2(v_2) \end{aligned}$$

And $(\sigma_1 \bar{\times} \sigma_2)(u_1, u_2) = \sigma_1(u_1) \vee \sigma_2(u_2)$, $(\sigma_1 \bar{\times} \sigma_2)(u_1, v_2) = \sigma_1(u_1) \vee \sigma_2(v_2)$

Thus $(\sigma_1 \bar{\times} \sigma_2)(u_1, u_2) \vee (\sigma_1 \bar{\times} \sigma_2)(u_1, v_2) = \sigma_1(u_1) \vee \sigma_2(u_2) \vee \sigma_2(v_2)$

Hence $(\mu_1 \bar{\times} \mu_2)((u_1, u_2), (u_1, v_2)) < (\sigma_1 \bar{\times} \sigma_2)(u_1, u_2) \vee (\sigma_1 \bar{\times} \sigma_2)(u_1, v_2)$

Therefore, $G_{A_1} \bar{\times} G_{A_2}$ is not a S-strong anti fuzzy graph which is contradiction. Hence $G_{A_1} \bar{\times} G_{A_2}$ is S-strong then atleast one of G_{A_1} or G_{A_2} must be S-strong.

Proposition 3.6

If anti composition of anti fuzzy graphs $G_{A_1} \bar{\circ} G_{A_2}$ is S-strong then at least G_{A_1} or G_{A_2} must be S-strong.

Theorem 3.7

Anti join of anti fuzzy graphs $G_{A_1} \bar{+} G_{A_2}$ is S-strong if and only if G_{A_1} and G_{A_2} are both S-strong.

Proof:

Let G_{A_1} and G_{A_2} be an anti fuzzy graph. By the theorem 3.5, consider that at least any one of G_{A_1} or G_{A_2} is S-strong then the anti cartesian product of anti fuzzy graph is S-strong. Which leads that $G_{A_1} \bar{+} G_{A_2}$ is S-strong.

Remarks:

1. The line graph of a anti fuzzy graph is always a S-strong anti fuzzy graph.
2. A full spanning anti fuzzy subgraph of a S-strong anti fuzzy graph is also a S-strong.

Theorem 3.8

(σ_1, μ_1) is a S-strong anti fuzzy graph of G_{A_1} and (σ_2, μ_2) is any anti fuzzy graph of G_{A_2} . $G_{A_1} \bar{\times} G_{A_2}$ is a S-strong anti fuzzy graph if and only of $\sigma_1(u_1) \geq \mu_2(u_2, v_2)$ for all $u_1 \in V(G_{A_1})$ and $(u_2, v_2) \in E(G_{A_2})$.

Proof:

Let $G_{A_1} \bar{\times} G_{A_2}$ be a S-strong anti fuzzy graph. Then for all $u_1 \in V(G_{A_1})$ and $(u_2, v_2) \in E(G_{A_2})$,

$$\begin{aligned} (\mu_1 \bar{\times} \mu_2)((u_1, u_2), (u_1, v_2)) &= (\sigma_1 \bar{\times} \sigma_2)(u_1, u_2) \vee (\sigma_1 \bar{\times} \sigma_2)(u_1, v_2) \\ &= \sigma_1(u_1) \vee \sigma_2(u_2) \vee \sigma_2(v_2) \longrightarrow \text{(iii)} \end{aligned}$$

By definition of anti cartesian product of anti fuzzy graph,

$$(\mu_1 \bar{\times} \mu_2)((u_1, u_2), (u_1, v_2)) = \sigma_1(u_1) \vee \mu_2(u_2, v_2) \longrightarrow \text{(iv)}$$

Therefore, from (iii) and (iv) We get,

$$\sigma_1(u_1) \vee \mu_2(u_2, v_2) = \sigma_1(u_1) \vee \sigma_2(u_2) \vee \sigma_2(v_2) \longrightarrow \text{(v)}$$

If $(u_2, v_2) \in E(G_{A_2})$ then we have $\mu_2(u_2, v_2) > \sigma_2(u_2) \vee \sigma_2(v_2) \longrightarrow \text{(vi)}$

From (v) and (vi) it follows that $\sigma_1(u_1) \geq \mu_2(u_2, v_2)$.

Converse part, let us assume that $\sigma_1(u_1) \geq \mu_2(u_2, v_2)$ for all $u_1 \in V(G_{A_1})$ and $(u_2, v_2) \in E(G_{A_2})$.

To prove that $G_{A_1} \bar{\times} G_{A_2}$ is a S-strong anti fuzzy graph.

we know that, $\mu_2(u_2, v_2) > \sigma_2(u_2) \vee \sigma_2(v_2)$

and $\sigma_1(u_1) \vee \sigma_2(u_2) \vee \sigma_2(v_2) = \sigma_1(u_1) \vee \mu_2(u_2, v_2)$ for all $u_1 \in V(G_{A_1})$ and $(u_2, v_2) \in E(G_{A_2})$.

For any other $(u_2, v_2) \in E(G_{A_2})$, $\mu_2(u_2, v_2) = \sigma_2(u_2) \vee \sigma_2(v_2)$ and so

$$\begin{aligned} (\mu_1 \bar{\times} \mu_2)((u_1, u_2), (u_1, v_2)) &= \sigma_1(u_1) \vee \sigma_2(u_2) \vee \sigma_2(v_2) \\ &= (\sigma_1 \bar{\times} \sigma_2)(u_1, u_2) \vee (\sigma_1 \bar{\times} \sigma_2)(u_1, v_2) \end{aligned}$$

Which shows that $(\mu_1 \bar{\times} \mu_2)((u_1, u_2), (u_1, v_2)) = (\sigma_1 \bar{\times} \sigma_2)(u_1, u_2) \vee (\sigma_1 \bar{\times} \sigma_2)(u_1, v_2)$

If $w_2 \in V_2$ and $(u_1, v_1) \in E(G_{A_1})$ then from the given G_{A_1} is S-strong which follows that

$$\begin{aligned} (\mu_1 \bar{\times} \mu_2)((u_1, u_2), (u_1, v_2)) &= \mu_1(u_1, v_1) \vee \sigma_2(u_2) \\ &= (\sigma_1 \bar{\times} \sigma_2)(u_1, w_2) \vee (\sigma_1 \bar{\times} \sigma_2)(v_1, w_2) \end{aligned}$$

From above all results shows that $G_{A_1} \bar{\times} G_{A_2}$ is a S-strong anti fuzzy graph.

IV. Application of S-Strong Concept in Agriculture

The S-Strong concept can be applied in the inheritance of pea seeds shape to obtain more round shaped pea seeds. The Pea shape can be round when it is associated with allele R or may be wrinkled when it is associated with allele r . Here allele refers to one variant of a gene on a chromosome. There are three possible combinations of alleles among which RR and rr are homozygous and Rr is heterozygous. The pea plants having RR chromosomes give rise to round peas and those with rr chromosomes will produce wrinkled peas. The plants with Rr chromosomes, the R allele masks the presence of the r allele. These plants may also produce round peas. Thus, allele R is completely dominant to allele r and is responsible for generating round peas whereas allele r is said to be recessive to allele R . By applying the S-Strong concept, we can prove that pea plants having atleast one R allele in the chromosome can produce Round Peas. If the breeding of pea plants having RR or Rr chromosomes is done in large levels, then it will increase the profit of the farmers significantly by producing more round

peas. Other fine instances where S-Strong concept can be applied are in the inheritance of snapdragon flower with red and white colors to produce flowers with color combinations as well as in the inheritance of white bulls with black cows to produce healthy offsprings.

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